Study of primary school teacher's practices for a lesson after a Lesson Study process in mathematics

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This study presents the analysis of primary school teacher's practices in mathematics for a lesson that has taken place after a professional development training called lesson study (LS) in Lausanne, Switzerland. Practices are analysed in a double didactical and ergonomical approach. The methodology used is a case study of the particular teacher's practices. Results about the teacher's practices after the LS process are discussed.

Keywords: Teacher's practices, problem solving, lesson study.

LS is a field of research and professional development developed principally in Asia, in US and in Northern Europe (Lewis & Hurd, 2011; Yoshida & Jackson, 2011). LS is a collective and reflexive process that involves a group of teachers and facilitators meeting to improve instruction.

This study concerns practices of a teacher who participated to a LS process and this study falls within the "double approach" (Robert & Rogalski, 2002, 2005).

Theoretical Framework: the "double approach"

Teachers' practices are analysed using the following theoretical framework: the "double approach" based on a French didactical approach (Robert & Rogalski, 2002, 2005) and an ergonomical approach based on activity theory (Leontiev, 1975; Leplat, 1997).

This "double approach" distinguishes the task, the "goal to be attained under certain circumstances" and the activity, what the teacher engages in during the completion of the task (Rogalski, 2013, p. 4). The prescribed work fits the *prescribed* task (in our context: what the teacher must do according to Teacher's Handbook and the official program) and the real work fits the *conducted* task (in our context: what the teacher does in reality during the lesson). To appropriate the *prescribed* task, the teacher should modify it. Thus, a gap exists between the *prescribed* task and the *conducted* task: the reasons can be a lack of the necessary competencies or an inappropriate representation of the task for example (Ibid.). Leplat (1997) adds two levels of tasks: the *represented* task (in our context: how the teacher represents the *prescribed* task and what he thinks we attend of him) and the *redefined* task (in our context: the teacher redefines his task according to the *prescribed* task and his own professional goals). These levels of tasks are neither hierarchical nor time: the teacher can represent the *prescribed* task and can redefine a new task before and during the lesson in taking into account different sources (students' activity, his own activity, institutional constraints).

The teacher combines professional acts and knowledge (mathematical, didactical, pedagogical) in his representation of the *prescribed* task and in his redefinition of the *represented* task. This study focused on these professional acts and knowledge at stake in these representation and redefinition. Thus, the teacher's activity is analysed as a process of modifications between the *prescribed* and *conducted* tasks (Leplat, 1997; Mangiante, 2007).

Research question

This case study aims to provide elements to respond to the following question: what are the sources of the process of modifications between the *prescribed* and *conducted* tasks?

Methodology and data

This qualitative study used a case study to analyse teacher's practices. A LS process (see Figure 1) can be decomposed four steps (Lewis & Hurd, 2011, p. 2): the group studies a mathematical subject, standards and sets instructional goals (step 1), the group prepares a research lesson based on their study of the topic and standards (step 2), the group selects one teacher to conduct the research lesson while others observe and collet student data (step 3), and finally, the group analyses and reflects on the research lesson (step 4), with the option of teaching it again (Batteau, 2016).

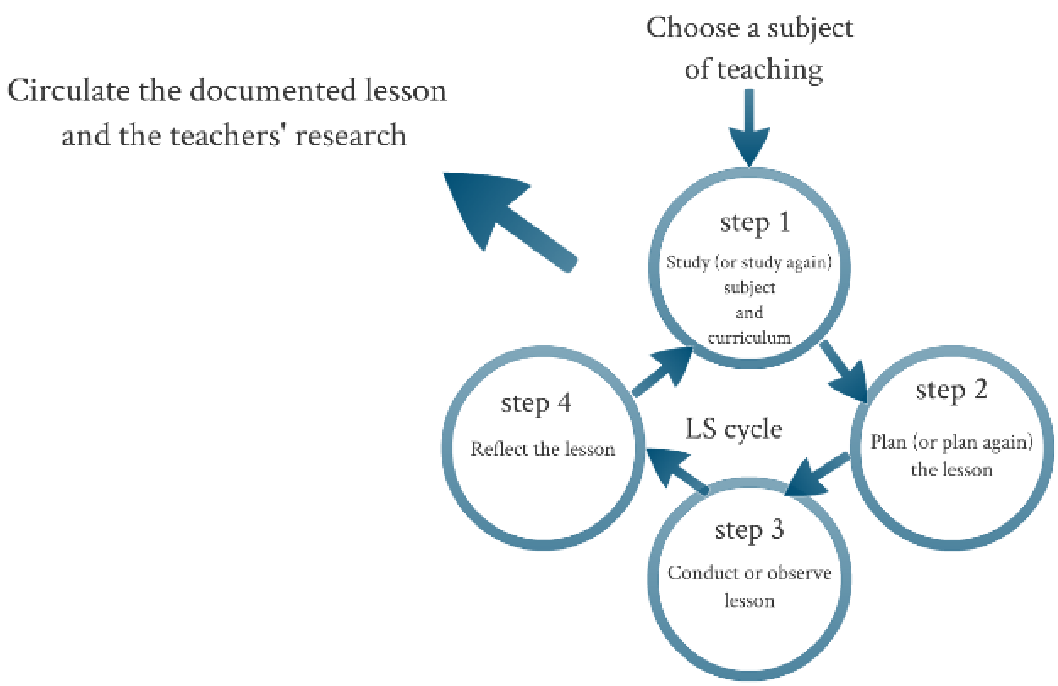

Figure 1: A LS cycle (Lewis & Hurd, 2011)

In the Swiss context, some researchers chose to implement this form of LS process without modify the structure in four steps because this model fits a French didactical point of view (Clivaz, 2015): the Theory of Didactical Situations (Brousseau, 1997; Warfield, 2014). In the TDS, the methodological research tool consists of an *a priori* analysis of the possible teaching of a mathematical subject: the steps 1 and 2 fit a deepen *a priori* analysis with a study of the mathematical subject, the didactical variables, the student's strategies, and difficulties. A second methodological research tool (in the TDS) consists of an *a posteriori* analysis, who takes place during the step 4, which compares what would be anticipated and what is happened. Thus, a LS process in mathematics began in Lausanne in September 2013 and occurred over two years with two collective sessions occurring per month (Clivaz, 2016). The group consisted of eight primary school teachers ranging in experience, voluntary, and generalist[1] teachers, and two facilitators[2].

This study focused on Océanes' practices for one lesson observed after the end of this LS process. For this teacher, data were collective sessions during the LS process (cycles *c* and *d* about problem solving), one lesson after the LS process (about problem solving), informal meetings after this lesson, all written documents produced during the lesson, and student work. Video data (lesson and collective sessions) were transcribed. This teacher has seventeen years of experience and students eight to ten years old.

To operationalise the theoretical tools presented for this study, the *prescribed* task fit the aim of the problem chosen by Océane, the Teacher's handbook, and the planning material for this problem. The *prescribed* task is analysed *a priori*, which means the mathematical knowledge at stake in the problem, the possible resolutions, and the didactical variables were analysed. We analysed the modifications between the *prescribed* and *conducted* tasks. To explain these modifications, we analyse the representation of the *prescribed* task and the redefinition of the *represented* task with using the informal meetings and collective sessions.

This paper presents first the mathematical subject worked during the LS cycles *c* and *d* before this lesson.

Analysis

Cycles *c* and *d* of the LS process

During the cycles *c* and *d*, the group worked on problem solving and how to help students represent a problem. The group relied on an article (Julo, 2002)[3] in which the main idea was explained during a collective session.

Facilitator: (quoting Julo) "this help doesn't give clues about the answer, doesn't guide to a strategy and doesn't suggest a modelling". But it's difficult to achieve, it's written just after that. It is an ideal […] but if we don't follow this ideal, it means that we do precisely a part of what students have difficulty to do.

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[1] In the French speaking part of Switzerland, primary school teachers teach several school disciplines (mathematics, French, sciences...).

[2] In this particular LS process, the two facilitators were researchers in Mathematics Education and in teaching and learning (Clerc-Georgy & Clivaz, 2016). They had the role of trainers and « knowledgeable others » (Lewis & Hurd, 2011, p. 30;33).

[3] During the first collective session of LS process, teachers said to facilitators which subject they wanted to work according to their teaching difficulties and/or students' difficulties. The subjects were numeration (cycle *a*), isometries (cycle *b*), and problem solving (especially how to help students represent and model a problem, cycles *c* and *d*). Then, the facilitators proposed reading this paper to teachers, in order to find elements of answer to this issue.

The research lesson of the cycle *d* was based on this problem: examine the matchstick pattern represented below. How many matchsticks are needed to align 99 squares?

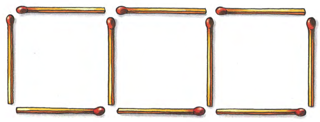

Figure 3: Problem "99 squares" (Teacher's handbook of 6H, Danalet, Dumas, Studer, & Villars-Kneubühler, 1999, p. 187)

The mathematical function at stake was u(n)=3n+1, where n is a whole number. The group worked on this problem focussing on how to help students represent and model this problem.

Context of the lesson observed after the LS process

For this lesson, Océane chose to manage problem solving and she explained it during a collective session at the end of the LS process.

Océane: There is a lot of problem which I think oh I don't dare to try [...]

Océane: I think, this year with my students, I take the textbook and I do a lot of things I never did before.

Anaïs: Oh, you dared.

Océane: Yeah, I did.

The *prescribed* task: some elements of analysis

For this lesson after LS, Océane chose the problem "Fold": Fold a strip of paper in half, here are two parts. Fold a strip of paper in half, then a second time, here are four parts and so on. How many parts are there with a folded strip of paper ten times?

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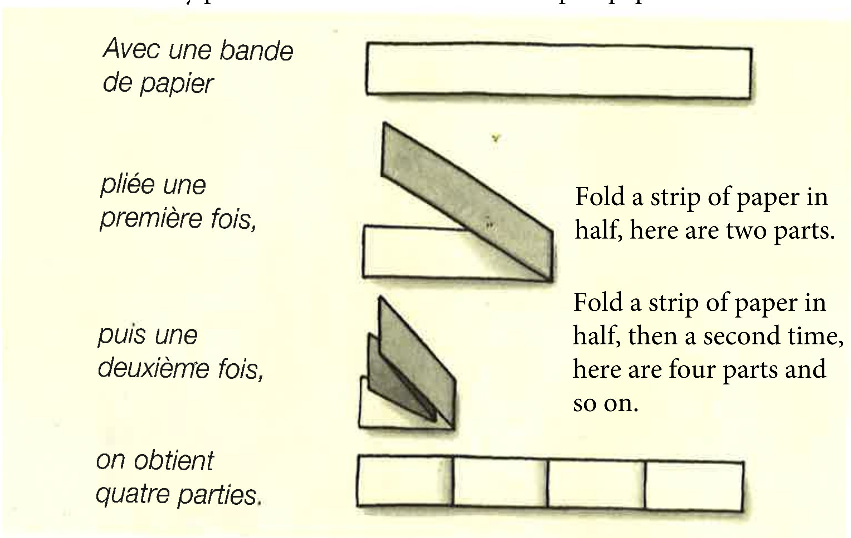


Figure 4: Problem "Fold" from (teacher's handbook, Danalet et al., 1998, p. 96)

The aim of this problem is to develop reasoning capacities and research strategies (Ibid.). In this problem, students should go from handling to representation in order to predict the result of acts (Ibid.). To determine the number of parts when the strip of paper is folded 10 times, we should calculate 2x...x2 with ten factors 2 (or 2 power of ten). Thus, to find the number of parts with a strip of paper folded *n* times, it's not necessary to know the answer when the strip of paper is folded (*n-1*) times.

The problem solving "Fold" is similar to "99 squares" in that sense these two problems rely on functions (power function or affine function). The kind of functions and the context of these problems are different but the idea of function is the same and the idea that it's possible to determine the number of matchsticks whatever the number of squares or the number of parts whatever the number of times we fold the strip of paper.

Modifications between the *prescribed* and *conducted* tasks

Some significant elements of modifications between the *prescribed* and *conducted* tasks are summarized about the mathematics at stake in the problem. Océane took over the modelling of the problem: she realized a two-column table, then students had to complete it by calculating doubles. Thus, she modified the aim of the problem. During informal meetings, she said that she chose this problem to introduce the multiplication. The issue of the problem is not the same for her (involving the multiplication) and for the designer of the problem (modelling a problem). During the lesson, she took over the modelling of this problem instead of students. Furthermore, she reduced the problem to calculations of doubles of numbers as in this characteristic extract of the lesson.

Teacher: doubles. Here, we double. We double every time. The double of two, four. The double of four, eight. The double of eight, sixteen. The double of sixteen, thirty-two. The double of thirty-two, sixty-four. The double of sixty-four? All right? So Nadège, the double of sixty-four is? It folds in seven times. […] It's as if we calculate sixty-four more sixty-four. Is it? (*Nadège looks all the folds in her strip of paper*).

Nadège: one hundred twenty-six. One hundred twenty-eight.

Teacher: great. […] Next, Luc?

Luc: two hundred fifty-six.

Teacher: very well. Yes? If we fold it nine times, it should be?

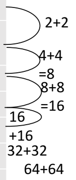
Romuald: five hundred six.

When Océane prepared her lesson, she did not identify the mathematical knowledge at stake in the problem (power of two). She validated students' strategies only with calculations (see extract), and she did not link strategies together. In this extract, she said "the double of sixty-four is? It folds in seven times". However, she did not explain why it's necessary to multiply by two when the strip of paper is folded half. Her strategy of doubling could not allow to respond directly to the problem. With her strategy of doubling, in order to find the number of parts with a strip of paper folded ten times, it's necessary to know the answer when the strip of paper is folded nine times and eight times, …, until two times (see Figure 5). With the "expert" strategy, to find the number of parts when the strip of paper is folded n times, the students should calculate the product 2x2x….x2 with n factors 2.

Using a similar problem solving activity than for the research lesson of the cycle *d*, Océane could not identify the mathematical function at stake.

Another modification of the *prescribed* task was to propose to students to calculate the number of parts when the strip of paper is folded 11, 12, 13, and 14 times (see Figure 5). This modification was coherent with the teacher's strategy because it was not possible to propose to calculate the number of parts when the strip of paper is fold 100 times for example without the "expert" strategy.

FOLD	PARTS
1	2
2	COUNT UP TO 4
3	8
4	16
5	32
6	64
7	128
8	256



9	512
10	1024
11	2048
12	4096
13	8092
14	16384

Figure 5: Reconstitution of the blackboard

In the blackboard, Océane wrote only additions to fill in the table, but nor multiplication neither "double of a number". To fill in the second line of the table, she wrote two strategies without linking: count up to 4 and 2+2.

This modification illustrated the focus of the lesson on calculation of double (with additions) and nor on modelling the problem, neither on the explanation of strategies and the links between the different strategies.

Representation of the *prescribed* task

Océane represented the *prescribed* task in according to her mathematical analyses. Before teaching, she prepared her lesson and realised mathematical analysis. The issue of the problem (modelling) took over by the teacher. In her analysis, the mathematical knowledge at stake in the problem are multiplication and doubling of a number. In the teacher's handbook, the aim is to represent, to model a problem, to develop reasoning capacities and research strategies. Her analysis was in contradiction with the teacher's handbook. Thus, she took freedom in relation to institutional constraints of the Teacher's textbook.

Redefinition of the *represented* task

Océane anticipated the two-column table to fill in, so she anticipated to take over the representation of the problem and his modelling before this lesson. During this lesson, she taught vocabulary, "double of", and she focused only on calculations. In her redefinition of the task, she modified the problem in a problem of calculation when the strip of paper is folded 2 until 14 times.

In her redefinition of the task, she modified the problem according to her mathematical analysis and her representation of the task.

Process of modification between the *prescribed* and *conducted* tasks

The process of modification between the *prescribed* and *conducted* tasks had its origins in her representation of the *prescribed* task for this lesson. Océane took into account the students' activity for the first time she taught this problem (last year). Then, she adapted her teaching when she taught this problem for the second time (for this lesson after LS): she took over the modelling and imposed a two-column table to fill in. She did not take into account students' activity during this lesson but by anticipation.

Conclusion

This case study proposed an analysis of particular teacher's practices during a lesson after a LS process. After the LS process, this teacher has self-confidence over teaching problem solving. In teaching problem solving, it should be able to identify mathematics at stake in the problem. Mathematics at stake should be given by the mathematical textbooks, but it was not the case. In the French part of Switzerland, official textbooks lack mathematical analysis for the teacher to use while planning lessons. For this lesson, the representation of the *prescribed* task relied on the Océane's mathematical analysis which were not sufficient. Thus, her representation and her redefinition of the *prescribed* task did not allow to reach the mathematical learning intended by this problem. To conclude, the sources of the process of modifications for this lesson were her representation of the *prescribed* task and her mathematical analysis. This case study highlighted a gap between the *prescribed* and *conducted* tasks due to the teacher's representation and mathematical analysis.

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