The role of affect in failure in mathematics at the university level: *the tertiary crisis*

Pietro Di Martino[1] and Francesca Gregorio[2]

[1]University of Pisa, Mathematics Department, Pisa, Italy; pietro.di.martino@unipi.it

[2]University of Pisa, Mathematics Department, Pisa, Italy; gregorio.francesca@gmail.com

The tertiary transition between secondary school and University appears to be an insurmountable struggle for many students. This is also the case, surprisingly, in a certain sense, of students enrolled in the Mathematics degree, and therefore students considered "gifted" with respect to mathematics. This case seems particularly interesting from an affective point of view: these students often live failure in mathematics as a tragedy, and – above all – initially they are not able to interpret their failure. For these reasons, it appears crucial to investigate which role is played by emotions in the arise and management of this crisis, and how the students' view of mathematics and their self-perception develop in the tertiary crisis period.

Keywords: Mathematics failure, affect, gifted student, theories of success in mathematics.

Introduction and theoretical background

As Nardi (2008) underlines, the teaching and learning of undergraduate mathematics is a relatively new field of mathematics education research.

The most part of the research about this topic focuses on cognitive aspects, highlighting the difficulties related to the learning of advanced mathematics (Artigue, 2001). In particular, Tall (1991) discusses the students' difficulties in conceptualizing some specific mathematical constructs (for example the notion of limit of a function), and in using the formal definitions of these constructs.

Other scholars focus on the specific difficulties related to the tertiary transition, discussing the enormous gap between secondary and tertiary mathematics (De Guzmàn et al., 1998; Wood, 2001) in terms of cognitive, metacognitive, linguistic and also practical demands.

Alcock and Simpson (2002, p. 33) underline how "certain reasoning strategies are inadequate when applied to university mathematics, although they might be efficient and sufficient in non-technical contexts and in the kind of reasoning with specific objects required by school mathematics". Schoenfeld (1985) analyzes the undergraduate students' difficulties in managing with non-routine tasks. Ferrari (2004) discusses the linguistic difficulties related to the shift from an informal approach to mathematics to a formal one. Scholars (De Guzmàn et al., p. 756-757) underline how "many students arriving at University do not know how to take notes during a lecture, how to read a textbook, how to plan for the study of a topic, which questions to ask themselves".

All the scholars describe the mathematical tertiary transition as a very challenging moment for students: Tall (1991, p. 25), points out that it "involves a struggle (...) and a direct confrontation with inevitable conflicts, which require resolution and reconstruction".

Nevertheless, affect appears to be the 'great absent' in this overview about the factors playing a role in this transition: for example, in the previous editions of CERME, there are no reports of the TWG about affect related to the undergraduate level (with the exception of the study related to the undergraduate course needed to become a primary teacher). This appears as an anomaly, because it appears evident as tertiary transition involves emotional aspects. In particular, Clark and Lovric (2008), drawing on anthropological theories, describe the tertiary transition in mathematics as a three-stages rite of passage that includes: separation (from secondary school), liminal (from secondary school to University) and incorporation (into University). This rite of passage is characterized by a real crisis in which the consolidated routines are suddenly interrupted, changed and distorted. As Bardelle and Di Martino (2012) underline, this crisis appears to be particularly challenging for mathematical high-achievers in the secondary school level: it can be difficult for these freshmen to understand why the reasoning strategies that worked in their previous mathematical experiences suddenly stop working at university level. In a recent paper, Rach and Heinze (2016) underline the need to consider also affective learning prerequisites in the tertiary transition. Basing on empirical evidences, they identify and study five individual variables that affect successful mathematical learning process at the university level: interest, self-concept, specific prior knowledge, prior achievement, and learning strategies.

Therefore, it becomes interesting to investigate how the high-achiever students' view of mathematics and their self-perception develop during the first phase of the transition and which role emotions play in the arise and management of this crisis. In other words, considering the TMA-model for attitude developed by Di Martino and Zan (2010), it is interesting to study the development of students' attitude towards mathematics during the *tertiary crisis*. With this aim, in the AY 2014/15 we developed a narrative study involving different categories of current and ex-students of the Bachelor in Mathematics in Pisa:

1) *freshman*: to collect the voice of the subjects during the crisis period;
2) *expert students*, i.e. students enrolled in the third year of the Bachelor: to understand what they remember about their transition difficulties, their idea about the causes of such difficulties and about how they overcame them;
3) *dropout students*, i.e. students that have left the Bachelor in Mathematics without obtain the degree in Math: to collect their memories about the crisis period, in order to reconstruct their emotions, the motives of their resignation, their theories of success (Nicholls et al., 1990) and causal attributions (Weiner, 1986).

Context and Methodology

Context The Bachelor in Mathematics in Pisa is one of the most prestigious in Italy, the majority of its students is considered excellent in math during secondary school. This fact is confirmed by data collected for our research. We analysed data from AY 2009/10 to 2012/13: Table 1 shows the percentage of the high-rated students in final exam of secondary school (we define high-rated a mark between 90/100 and 100/100). It compares the situation in Pisa with the average from all Italian Bachelors in Mathematics.

	2009/10	2010/11	2011/12	2012/13
Italy	45%	42.4%	44.5%	40.1%
Pisa	73.6%	58.6%	65.7%	60%

Table 1: Percentage of high-rated students in the Bachelor in Mathematics, in Pisa and nationalwide.

The percentage of high-rated students in Pisa is much higher than the global one. However, the dropout rates of the Bachelor in Pisa are within the national average range. The high concentration of above the average students and the presence of difficulties, as witnessed by the failure rates, make the Bachelor in Mathematics in Pisa the ideal contest for our research.

Procedure The study was conducted in two different phases. In the first phase, we developed and administered three online questionnaires (one for each category of the involved students) including open and close questions about the mathematical experience at the University and, in particular, the difficulties encountered. Students were requested to answers in an anonymous way: respondents were invited to share an e-mail address in order to participate in the second, non-anonymous, phase of the research. The participation to this study was voluntary. At the end of this first phase we had collected: 26 answers by freshmen; 75 by students enrolled in the third year of the Bachelor; 52 by students that had left the Bachelor.

In the second phase, 40 students (3 freshmen, 27 *expert students*, 10 *dropout students*) were interviewed by the second author of this report. The time for the interviews varied in a range from 5 to 90 minutes. The interviews were audio-recorded and then fully transcribed.

We will quote the students' answers using an alphanumeric code: F, E or D (which mean freshman, expert student or dropout student, respectively); a serial number (it indicates the order in which the student completed the questionnaire); Q and I (which mean questionnaire and interview, respectively).

Rationale We developed a narrative approach because we wanted students to feel free to express what they consider important, using the words that they consider more appropriate. In particular, we considered the open-ended questionnaire and the interview to be two complementary narrative instruments: according to Cohen et al. (2007), an open-ended question can catch the authenticity, richness, depth of response, honesty and candor which are the hallmarks of qualitative data. On the other hand, questionnaires have their limitations: they are one-way compared with interviews.

Results and discussion

In the students' stories the difficulties are often linked to strong and negative emotions, persisting over time. Both questionnaire and interview had specific questions about emotions; for example, a question in the questionnaire was: "*Write a feeling that is linked to your experience at the Bachelor in Mathematics*". E69Q, despite he was able to overcome the initial difficulties, reports: "*unfortunately, now* [after having dropped out of the Bachelor in Mathematics] *I like math a lot less, or rather, it still fascinates me but it is now linked to very negative emotions that ruin it all*".

The percentage of students who indicate negative emotions changes drastically depending on the category of interviewed students. Only 32% of the freshmen report having bad feelings relating to their experience at the Bachelor in Mathematics and to their difficulties and failures. This fact may be connected with some characteristics of the sample: the questionnaire was published at the end of the academic year, when the students with serious difficulties have already left the Bachelor. The percentage of students that report having bad feelings increases among the *expert students*: as many as 52% write about bad feelings and emotions. Predictably, the percentage increases among the *dropout students*: 75% of these students link difficulties and the experience at the Bachelor in Mathematics with very negative feelings. Different types of bad feelings are reported, we have identified some categories: anxiety/ distress/ anguish; frustration/ despondency/ hopelessness; fear/ apprehension; sadness/ sorrow/ depression; inadequacy/ insecurity.

The majority of bad feelings are related to the anxiety caused by the Bachelor in Mathematics: "*for the first two years I was in a permanent state of anxiety and distress*" (E6Q), "[my experience in Pisa has been] *angsty*" (E8I). This topic is reported by all three categories of students.

The frustration and despondency category appears despondency among the *expert students*, but also among the dropout students. Many students report they are not been able to reach their goals via techniques and mechanisms that had been successful in the recent past. The persistence of such a situation forces many students in a state of frustration and it brings them to reevaluate their skills: "*I realized that I would go to class and not understand a word of what was being explained. Therefore, I felt some frustration and I thought that I was not intelligent enough, that I was inadequate*" (D49Q).

The category of sadness and sorrow characterizes particularly the students who left the Bachelor in Mathematics. The decision of leaving the Bachelor in Mathematics seems to be linked to a strong sadness: "*I remember* [of that period] *just a lot of tears*" (D39Q).

Also the category of inadequacy and insecurity is strongly linked to the experience of difficulties, and in particular to their lasting and to the failure in overcoming them. These feelings are new for the students, they never felt them before because they have always been good at math. The consequences, in these cases, can affect the students' self-esteem and his or her learning abilities: "*I think that my experience at the Bachelor in mathematics left me with less confidence in my ability to study*" (D10Q), "*as far as I'm concerned, low self-esteem kills any productive drive*" (E8Q).

The emotions reported by the interviewed are often felt as negative because they are unexpected: the student shifts suddenly and in unexpected ways from a *mathematical welfare* to a *mathematical malaise*, and he doesn't understand the causes of the shifts. The persistence of the difficulties causes the growth of bad feelings and the sense of helplessness ("*there were a lot of difficulties and disappointment… I felt like there was no way out*", E29Q), contributing to foster a downward spiral.

In our study a special attention has been given to the students' attributions for their difficulties. The students both spontaneously or answering specific questions, have made causal attributions. The narrative data collected have permitted us to identify the more frequent causal attributions, and to organize them in categories:

- *Transition aspects*: differences between secondary school and University (contents, organization, teaching styles, assessment, …);
- *Low preparation*: insufficient secondary school prerequisites;
- *Low ability*: lower math ability than they thought, inadequate mindset (these factors are often attributed to a faulty way to assess in secondary school)*;*
- *Comparison aspects*: many of these students were considered (and perhaps they really were) the best math-students in their school, and for the first time in their life they are "one of many". This impacts with their self-perception in math.

The students blame an important part of their failures to the great differences (related to math) between secondary school and University. These differences and the subsequent difficulties often cause significant changes in the students' view of math, and in particular of what it means to be good in math. In particular, most of the students point out that they got good grades in secondary school without significant efforts: "[in secondary school I considered myself good at math] *because I could avoid studying it and still get the highest grades*" (F14Q) and "*I realized that the high learning speed was due to the easiness of the topics we studied in high school, rather than to an above the average skill*" (D51Q). The secondary school's math is regarded as a simplified and procedural math, surely not as the *math* studied at the University: they seem two completely different subjects! ("*Math you do in high school is not the one you do in your first year in University*", E52Q). Students recognize that in University more formalism, abstraction and proofs are required: math switches from numbers and figures (a practical mathematics) to structures, like vector spaces or groups (a theoretical mathematics), and it involves a radical and hard cognitive shift. Despite the connected difficulties, the discovery of this new math usually is welcomed ("*I think I like the subject even more than I did in high school. I've found topics that I find fascinating*", E65Q), but sometimes it isn't ("*I've changed Bachelor since I couldn't find the practical math I was expecting*", D6Q). Anyway, the crucial point seems to be that this *discontinuity* in the subject is typically unexpected by the students: they choose Mathematics with a clear idea of what it is and of how much they are good in math, and suddenly they have to compare with a new reality.

Among the transition aspects, teachers and style of teaching have a predominant role. There is a shared perception amongst students that at the university level there is not a particular attention to the students' difficulties: it is interesting to underline that this perception is often shared also at secondary school level by students with difficulties in math (Di Martino & Zan, 2010).

Students also underline the fact they are left alone from the beginning: "*in University they gave for granted many notions, or they didn't focus enough on topics they deemed to be easy, creating enormous doubts and flaws*" (D41Q). So students feel abandoned and powerless against apparently insurmountable difficulties, unable to find successful strategies.

From our data, it emerges that students blame responsibility to secondary and university teachers for their transition difficulties. In their view, secondary school teachers did not teach them what math really is and how it needs to be studied, and university teachers do not pay attention to the natural difficulties in the transition.

In this framework, math is seen (often for the first time for these students) as intrinsically complicated, and the transition aspects seem to add up further difficulties. Many students thought, and continue to think, a particular mindset is required to succeed in math, and this “math mindset” is innate (“*from birth you are not cut out for it, as you would need to be*”, E45I).

The great amount of difficulties in the transition and this belief represent an explosive mix: according to the students’ narrations, it is one of the main causes of resignation (to be good in math you need to have an innate talent; now, with the real math, I’m not good in math; so I don’t have the innate talent and I can’t do anything to improve, because I’m not talented).

The above explosive mix is also strongly affected by the comparison with peers. Most of the interviewed were the best of their class, or even of their school, during secondary school. At University, the context is completely different: you are one of many, and – above all – there is a natural reluctance in sharing personal difficulties (this reluctance appears to be linked to the emotional reactions to the difficulties we have commented before). The consequence is the spread of a feeling of loneliness, a lot of students stated that they thought to be the only ones in that context with difficulties: they believed that most of their peers understood all without difficulties. This (wrong) perception affects and quickens the change in the math related self-perception of the students, creating doubts about their own brightness. This has strong effects on the emotional side: “*I have really downsized the opinion of myself I had by seeing that there were way more capable people than myself*” (D33Q), “*I had begun to think that maybe I wasn’t so good as I had thought and that it had all been an illusion. Moreover, I saw geniuses that new everything and understood everything right away and so I felt like an idiot*” (D44Q).

Despite a lot of common themes, there are also some significant differences between the causal attributions for difficulties of the expert students and those of the dropout students. In particular, most of the dropout students claimed that, despite a hard and extensive study, or even despite the supposed sufficient comprehension of math, they failed the exams. In their opinion, the reasons for the lack of success is therefore linked to their natural inability or even stupidity: they seem to think that a kind of innate ability is needed to succeed in mathematics. Other respondents said that one of the reasons for failure was that some professors seemed to teach only for the excellent students, without taking care of the bulk of the average students. So, the exceptional students’ presence is seen as a problem, as much as the exams’ scale of evaluation.

In the final analysis, the dropout students used especially external and uncontrollable causal attributions. Also the freshmen and the expert students used external causal attributions but, they reported that after of an initial period in which the difficulties were perceived as uncontrollable, they found a way to turn them in to something controllable. In particular, they refer to a shift in their theories of success or to a change in the strategy they adopted to deal with their pre-existing theories of success.

The students report of some strategies or changes that have led them to the overcoming of their difficulties; the most frequent reasons are relative to the quantity and quality of their study and relative to their study habits. A lot of the interviewed spoke about the cooperation with peers as of being of great help: “*obviously a relevant part of my success is due to the people that have* *supported me*" (E44Q) and "*Personally it was group study that allowed me to go on*" (E7Q). Peers, especially better students, also helped to find the right study habits and to create the necessary mindset to succeed in math: "*the older students helped me by convincing me that it was all about getting settled with new ways of reasoning*" (E44Q).

So, as a student said, "*challenging one's study habits*" (S63I) is important; the first step is to understand that the study habits are amendable and this happens especially after failures and through the comparison with peers or teachers. From this quote, as from many others, it appears that the personal awareness of what is going wrong and what can be improved is a necessary step towards overcoming one's difficulties. "*Seeing the teachers in action has been fundamental for me, in the sense that it helped me adopt the right mindset. By just studying on the books, I would have never obtained the same results*" (E52Q). Teachers are also fundamental for their emotional support: "*some teachers were fundamental in the process of overcoming my difficulties! In my opinion it is important that the professor lets you know that he believes in you, that he is aware you spent months preparing for the exam, that he is sorry if he fails you and that you are not just a number!*" (E28Q). Moreover, lots of students have found the meetings with peers or teachers very useful, overcoming the fear of the professor's judgment, which is instead very common in secondary school. Finally, great study and effort are necessary: "*I overcame my difficulties by endeavor and maximum commitment*" (E44Q).

Conclusions

From our study there thus emerges a path that seems to characterize the experience in the Bachelor in Mathematics. A student which was a high-achiever in high school enrolls in the Bachelor in Mathematics; almost always, in an unexpected and abrupt manner, he faces difficulties; these difficulties are linked to strong negative emotions (such as anxiety, frustration, sadness…) and are combined with a reevaluation of the previous scholastic experience and of one's skill in math; math is seen under a new light: it is, in some sense, new and it is taught differently; the student produces theories of success and causal attributions: these can be internal or external, but the difficulties are initially almost always perceived as uncontrollable.

Up to this point in the student's path, most of the stories we have heard agree, regardless of the interviewed student's category. But from now on there is a definite distinction between the experience of who has abandoned the Bachelor in Mathematics and who has succeeded in continuing his or her studies. The comparison of the experiences of the subjects from different categories has in fact provided us with precious information: those who, for possibly emotional reasons, persevere in producing uncontrollable causal attributions or in implementing the same strategies to reach success, will eventually drop out of the Bachelor in Mathematics; on the other hand, changing one's theories of success or one's causal attributions or just identifying them as controllable allows one to overcome difficulties and failure. Our study seems to suggest that what makes the difference between dropping out and overcoming the difficulties are one's success theories and causal attributions, and in particular the ability to modify them and identify controllable factors.

It thus seems that the processes that leed to changes in the students' success theories and causal attributions, which bring to light the controllable aspects of one's difficulties, is worthy of a deep investigation.

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