

# Types of talk and mathematical knowledge for teaching problem-solving: Dialogic analysis of a lesson study

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*This study investigates mathematical knowledge for teaching problem-solving in lesson study among primary school teachers. Eight meetings were analysed, revealing mainly three types of talk, cumulative, disputational, and exploratory, evolving across the lesson study cycle. Knowledge levels differed among the types of talk, offering insights into teacher knowledge development.*

*Keywords: Type of talk, lesson study, mathematical knowledge for teaching problem-solving.*

## Introduction

This research aims to analyse the type of mathematics problem-solving knowledge for teaching used when primary school teachers work collaboratively in a lesson study process and examine how dialogic interactions contribute to knowledge construction.

Two papers and one conference paper have already been published about the first part of the study. The first paper outlines the theoretical and methodological aspects (Clivaz, Daina, et al., 2023), while the second presents the results of the analysis of the first part of the LS cycle (covering five out of eight meetings, Clivaz, Batteau, et al., 2023). Following the CERME13 conference paper (Clivaz, Presutti, et al., 2023), this contribution links the micro-level utterances' dialogic features to types of speech at the meso-level of longer episodes during LS meetings. These results have not been published yet.

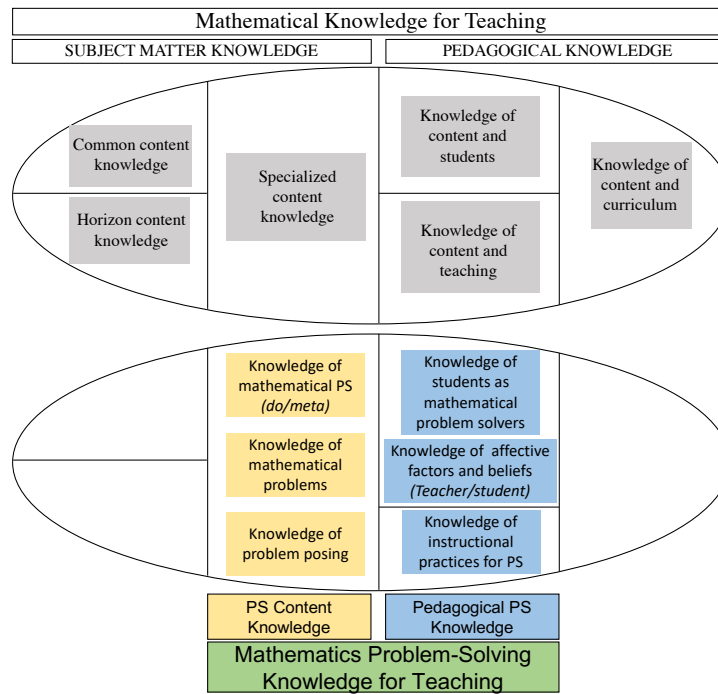
## Theoretical framework

This article lacks the necessary length to comprehensively expound upon the associated research's theoretical framework. Consequently, we direct the reader to our previously published works elucidating the concepts of Lesson Study (Clivaz, Batteau, et al., 2023, p. 2); knowledge levels (Clivaz, Batteau, et al., 2023, pp. 2–4); as well as Lesson Study Dialogic Analysis (LSDA, Clivaz, Daina, et al., 2023, pp. 24–28; 37). We present briefly here the Mathematical Knowledge for Teaching Problem-Solving, the Lesson Study Dialogic Analysis, and its link to the types of talk.

## Mathematical Knowledge for Teaching Problem-Solving (MKTPS)

Research on problem-solving (PS) has mainly focused on students' perspective. However, some studies have explored teachers' understanding of problem-solving and their mathematical knowledge for teaching (Ball et al., 2008). Based on a literature review of studies from 1922 to 2013, Chapman (2015) identified six categories of problem-solving knowledge for teaching, including problem-solving content knowledge and pedagogical problem-solving knowledge. These categories are influenced by teachers' problem-solving proficiency, affective factors, and beliefs. Adopting Chapman's findings, we propose a graphical representation of this categorisation (Figure 1) to bridge

the categories of mathematics problem-solving knowledge for teaching and mathematical knowledge for teaching.



**Figure 1: Mathematical Knowledge for Teaching Problem-Solving (MKTPS).**

**Mathematical knowledge for teaching** (the upper grey part of the figure) is taken from Ball et al. (2008); **mathematics problem-solving knowledge for teaching** (coloured categories) is from Chapman (2015). For the description of the categories, refer to (Clivaz, Daina, et al., 2023).

We then needed to determine knowledge levels (Table 1) to identify the participants' knowledge evolution throughout the meetings or differences related to the roles (teacher, facilitator). These levels do not constitute a hierarchy: contextualised knowledge or questioning is as valuable as generalised knowledge. The knowledge levels are labelled from 1 to 5.

**Table 1: Knowledge levels (Clivaz, Daina, et al., 2023)**

Knowledge levels	
1	Inaccurate knowledge, lack of knowledge, self-assumed ignorance, and/or debatable personal representation
2	Unexplained knowledge. Observation, testimony
3	Incomplete knowledge, knowledge with a low degree of certainty. Explicit questioning
4	Contextualised explicit knowledge. Speaker knows or appears to know the rationale
5	Generalised, decontextualised knowledge (decontextualisation process, possibly not fully completed). Generic example

### Types of talk

Our analysis of the utterances and types of talk is based on the work of the Cambridge Educational Dialogue Research (CEDiR) team who developed a Scheme for Educational Dialogue Analysis particularly the SEDA group (SEDA, Hennessy et al., 2016). We adapted their micro-level utterance

analysis to our context (Clivaz, Daina, et al., 2023). Following the SEDA method, we linked this micro-level analysis at the utterance level to the meso-level, defined by a series of speaking turns in which the topic remains constant. At this level, we adopted the three types of speech characterised by Mercer and colleagues (Littleton & Mercer, 2013, pp. 15-16; Wegerif, 2020, pp. 28–29):

**Cumulative talk:** Characterised by speakers building positively but uncritically on what the others have said. Everyone expresses their knowledge in the conversation, but there is no critical position or evaluation. Cumulative discourse is characterised by repetitions, confirmations and elaborations.

**Disputational /qualificatory talk:** Characterised by disagreements between discussants and a competitive atmosphere in which each person makes their own decisions. There is little sharing of resources or constructive criticism, and interactions are often about taking short positions without justification.

**Exploratory talk:** Characterised by everyone engaging critically but constructively in the conversation. Everyone brings information they consider relevant; this is questioned and argued, and group members seek to reach agreement before moving on to another stage. It is possible for an outside observer to follow the line of reasoning.

These types of talk have been determined by our LSDA indicators (Clivaz, Daina, et al., 2023, p. 37). In order to move from the classroom to the LS context, the members of the CEDiR group made some adjustments to these types of talk (Dudley, 2013). We also added the qualifier *qualificatory* to disputational and considered two more types of talk: **Structuring conversation** and **managing understanding**. These two types were less prevalent in our results, and we don't discuss them in this paper.

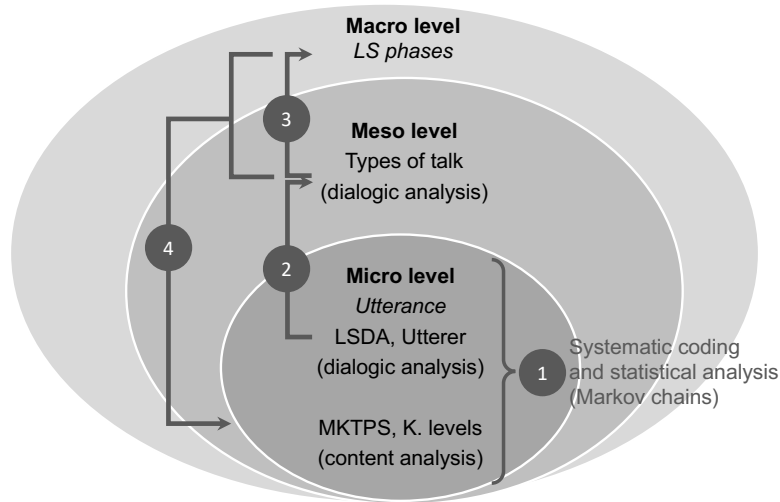
These types of discourse can be seen as different ways of thinking together. The exploratory talk is considered the most 'educationally effective' (Littleton & Mercer, 2013, p. 16).

## **Data, method and research question**

### **Context, data collection and method**

A LS group of ten teachers and facilitators from the Lausanne region was formed for this study. The facilitators, one math educator (the first author of this paper) and one teacher, aimed to focus on problem-solving in the classroom, without necessarily focusing on a particular mathematical topic. After discussing difficulties in teaching problem-solving, the group concentrated on helping students understand the problem representation. During the next meeting, they selected a problem from a state test that students struggled with in the past. The group then discussed representation, student approaches, and mathematics involved before planning a research lesson. After teaching and observing the lesson, the group discussed their observations of student performance and the importance of manipulatives, whole-class discussions, mathematics, and support for struggling students. Based on this discussion, they planned a second lesson in a second class, observed and discussed it. The last meeting allowed a discussion about the two lessons, the finalisation of the lesson plan, the assessment of the cycle and the desire for the next cycle.

The eight meetings were video recorded, transcribed and coded with the help of qualitative data analysis software. The coding of the data and the method are described in detail in Clivaz, Daina, et al. (2023). The three levels of analysis are nested, and provide a systematic structure for the analysis of the data. Systematic coding is carried out at micro-level. Then, the aim is to characterise the communicative events in such a way as to highlight the “interaction function” in the sense of Dudley (2013). Therefore, each communicative event can be characterised as a type of talk (arrow 2 in Figure 2). This type of discourse allows analysing the LS phases (arrow 3 in Figure 2).



**Figure 2: Nesting of analysis levels**

### Research question

Our main research question is: In what way is MKTPS built collectively during the LS process? For this article, we will focus on the sub-question: “What are the types of talk? Are they evolving? Is the MKTPS building related to the type of talk (arrow 4 in Figure 2)?”

### Results

#### Types of talk during the LS meetings

The various types of talk can be associated with specific phases within the LS cycle, the types of MKTPS, and the knowledge levels. Our initial hypothesis posited that the specific MKPTS category would not correlate significantly with the types of talk, and our analysis validated this assertion. We also anticipated that the types of talk would undergo evolutionary changes during the course of meetings, contingent upon the specific nature and focus of each meeting. Furthermore, we expected that certain levels of knowledge would manifest more prominently within specific types of talk. The ensuing sections present the findings about these two dimensions.

Figure 3 illustrates the predominant discourse modalities observed during the meetings: exploratory talk (53%), cumulative talk (23%), and disputational talk (10%). A closer examination of individual meetings reveals distinctive characteristics. For example, Meeting 1 (M1), a general discussion about problem-solving teaching, is distinguished by the prevalence of cumulative talk (58%). In this meeting, participants expressed their ideas and questions independently, with limited cross-

referencing or interdependence in their utterances. This contrasts significantly with M8, a discussion about the two lessons, lesson plan, cycle assessment, and considerations for the next cycle. In M8, participants exhibited a marked reduction in cumulative talk and a notable surge in exploratory talk (70%) and disputational/qualificatory talk (24%, both highest incidence within the LS cycle). During this meeting, participants actively engaged in building upon each other's ideas and occasionally expressed disagreements with other participants. This escalation in exploratory discourse is also conspicuous when comparing meetings with similar themes. M2 and M3, (study of the curriculum material) witnessed an increase in exploratory discourse during the latter meeting, transitioning from 37% to 62%. Similarly, the planning of the second research lesson (M6) was characterised by a predominant presence of exploratory talk (76%) compared to the planning of the first research lesson (M4), with a lower incidence at 50%. The contrast is even more pronounced when comparing the post-lesson discussion following the first research lesson (M5) with the one following the second research lesson (M6). In the first discussion, each teacher successively reported their observations, which were predominantly cumulative (48%). In contrast, during the second discussion, almost every observation was related to those of colleagues, comparing or contrasting the observations in an exploratory way (65%), or sometimes disagreeing in a disputational/qualificatory way (14%).

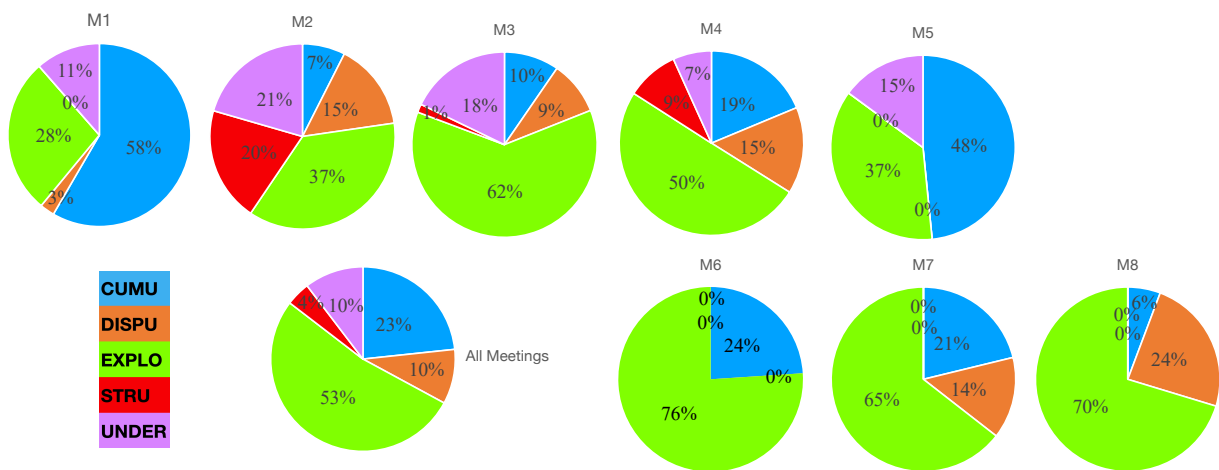


Figure 3: Type of talk for each meeting and for all the meetings

### Levels of knowledge present in the types of talk

Upon examining the knowledge level in the three primary types of talk, a distinction emerges between cumulative talk and exploratory talk (see Figure 4). Specifically, there is a higher prevalence of level 4 (Contextualised explicit knowledge. Speaker knows or appears to know the rationale) in exploratory talk (29% compared to 19%). In contrast, level 2 (Unexplained knowledge. Observation, testimony) is relatively less prominent. This difference can be attributed to the definitions of type of talk and knowledge levels. But it can also be rationalised by the observation that during phases of a meeting characterised by exploratory discourse (at the meso level), knowledge tends to be more comprehensively justified (at the micro level). The observation that disputation talk is close to exploratory talk in terms of levels of knowledge is more unexpected. However, a closer examination of the transcripts reveals that, despite taking “short positions without justification” in response to

previous utterances, participants in disputation talk often demonstrate that they know the rationale of this knowledge, with a reduced occurrence of unexplained knowledge compared to cumulative talk.

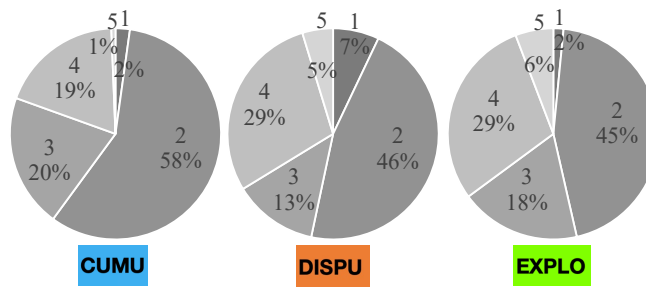


Figure 4: Knowledge levels for the three main types of talk

We refined our analyses by studying the transitions between levels of knowledge and this by type of talk (Figure 5).

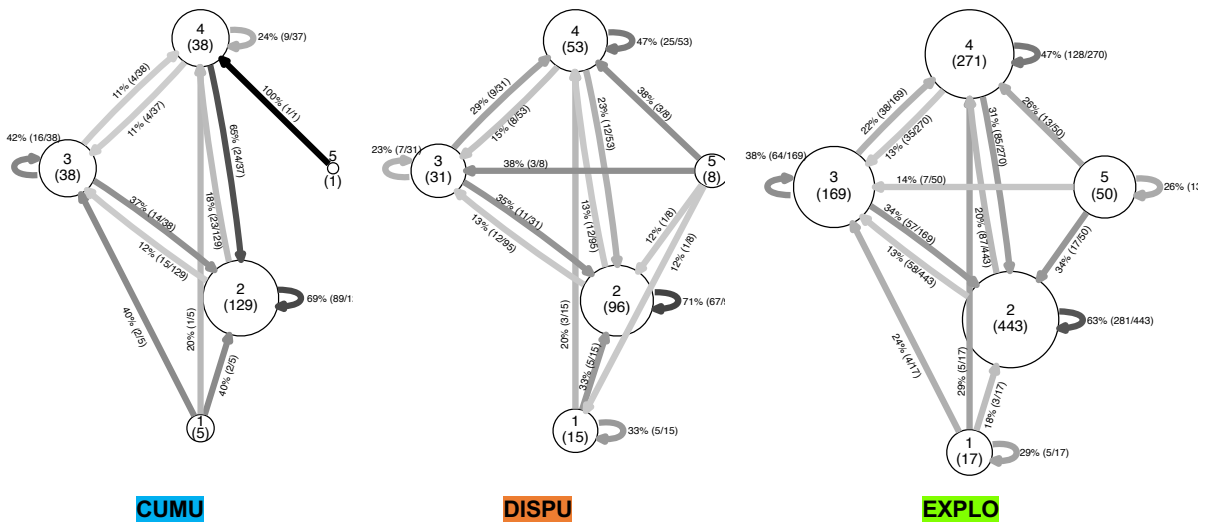


Figure 5. Markov chains for the levels of knowledge by type of talk

These Markov chains show that the exploratory and cumulative talk differ on the transitions from level 3 to level 4, going from 22% for the exploratory talk to 11% for the cumulative. This result can be interpreted as follows: when one speaker expresses incomplete knowledge or a question, the next speaker will express explicit contextualised knowledge. Similarly, the transition from level 4 to level 2 (unexplained knowledge, of the order of observation or testimony) is much more significant in the cumulative type of talk (65%) than in the exploratory type (31%).

These results from analyses of transitions between levels of knowledge illustrate how knowledge is constructed in exploratory talk in contrast to cumulative talk. These results confirm our research hypothesis based on the work of Littleton and Mercer (2013, p. 16) according to which the exploratory talk was judged to be the most "educationally effective". These results also allow us to analyse the dynamics in the construction of knowledge, through transitions between levels of knowledge.

In addition, there is no level 5 generalised, decontextualised knowledge in the cumulative talk, unlike the exploratory and disputational one. The transitions from levels 2, 3 and 4 to level 5 knowledge are all below 10% for the three types of talk. There is therefore no significant transition to level 5 of decontextualised knowledge, which we might have expected in the exploratory talk. On the other hand, there were transitions from level 5 to level 2 for the exploratory (34%) and disputational (12%) talk. Similarly, there were transitions from level 5 to level 3 for the exploratory (14%) and disputational (38%) talk, and to level 4 for the exploratory (26%) and disputational (38%) talk. As the number of occurrences is low, we can emphasise that in the exploratory and disputational talk, the decontextualised knowledge mobilised in one round is reexamined or recontextualised to classroom situations in the next round. These results illustrate the dynamics between decontextualised knowledge and knowledge of the order of questioning or classroom observation.

Level 2 knowledge is most present in the cumulative talk, in contrast to the other two. In a cumulative talk, participants express their knowledge without taking a critical or evaluative stance, and this knowledge is mainly level 2. In this type of talk, the participants talk about what they observed during the research lessons or refer to their classroom practices without considering the previous interaction.

The types of cumulative and exploratory talk are similar in the transition from level 2 to level 4, at 18% and 20%, respectively. Following a classroom observation, the turn to speak carries contextualised knowledge. This can be explained by the search for consensus in these two types of talk (Littleton & Mercer, 2013).

The transitions from levels 2 to 3 (12 to 13%) and 3 to 2 (34 to 37%) are similar for the three types of talk. This means that the type of talk do not differ with regard to the transition of knowledge linked to a class observation that is followed or preceded by a question or incomplete knowledge. Similarly, the transitions from level 4 to level 3 (between 11% and 15%) are similar for the three types of talk. These results may be explained by the very nature of the level 3 knowledge code, which involves either questioning, which may be conducive to knowledge building, or incomplete knowledge, which may be less conducive to knowledge building if it is not followed by level 4 or 5 knowledge.

## **Conclusions and perspectives**

The development of teachers' mathematical knowledge about the teaching of problem-solving in a collaborative setting is a complex process. The findings of this case study are specific to the group under examination, and individual factors have impacted the results. Nevertheless, the mixed-method approach provides a comprehensive and interactive perspective on the collective construction of MKTPS. The unique characteristics of LS and MKTPS stressed in our results reinforce the idea that LS and PS teaching are interdependent and demonstrate how teachers' professional knowledge is shaped through dialogic engagement during LS. This supports the notion that LS and PS teaching are "two wheels of a cart" (Fujii, 2018) and highlights the significance of a dialogic process in developing teachers' professional knowledge.

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