How do dialogic interactions contribute to the construction of teachers' mathematical problem-solving knowledge? Construction of a conceptual framework.

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Abstract

Purpose: The article presents the construction of a conceptual framework, which is rooted in mathematics education and in dialogic analysis. It aims to analyse how dialogic interactions contribute to constructing teachers' mathematical problem-solving knowledge. The article provides one example of this analysis.

Design/methodology/approach: The networking between a content analysis framework (Mathematical Knowledge for Teaching Problem-Solving) and a dialogic analysis framework (Lesson Study Dialogue Analysis) is presented. This leads to the construction of indicators to quantitatively and qualitatively code our data: five meetings during one lesson study cycle of a group of eight Swiss primary teachers, working on the teaching of problem-solving.

Findings: This article does not present empirical findings. The developed conceptual framework is the result presented.

Research limitations/implications: The presented framework allows modelling, on the one hand, the knowledge relating to the teaching and learning of problem-solving and, on the other hand, the analysis of interactions during a lesson study. The article does not contain the results of the research.

Practical implications: The use of our framework can contribute to teacher educators' and facilitators' training by highlighting which types of intervention are favourable to the development of knowledge.

Originality/value: Our analysis involves a "systematic coding" approach. It allows a fine-grained analysis of the interactions in relation to the evolution of knowledge. Such a systematic approach offers the possibility of questioning the coded data in various ways.

Keywords: teachers' mathematical knowledge, problem-solving, dialogic analysis, lesson study

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Introduction

The collective work of teachers, accompanied by mathematics researchers and trainers, is currently of interest to the international mathematics didactics community (Borko & Potari, 2020). Due to its historical roots in Japan and its many adaptations at international level, lesson study (LS[1]) training and research process is one of the systems at the centre of this interest. The process aims to contribute to the professional development of teachers (Murata, 2011) and, in particular, to the construction and evolution of their mathematical knowledge for teaching (Clivaz & Ni Shuilleabhain, 2019). The mathematical knowledge needed for teaching has attracted much attention in the mathematics teaching community (see, e.g., the report by Ball, 2017). One reason for this interest is the assumption that there is a relationship between teachers' knowledge, the quality of their teaching and subsequent student learning.

Therefore, this research aims to understand how mathematical knowledge for teaching is constructed and evolves during a lesson study. With this objective in mind, we are currently analysing the work of a LS group, composed of eight teachers of grades 3 and 4 from the Lausanne region (French-speaking part of Switzerland) and two facilitators. The two facilitators were a mathematics educator (first author of this paper) and a teacher from the school, who had previously been a member of another LS group in mathematics. From 2018 to 2019, this group carried out two LS cycles devoted to teaching problem-solving. Our research focuses on the beginning of the first cycle, which consisted of five meetings of a duration of around 90 minutes and one research lesson. Over the course of this research, we aim, through a detailed analysis of the interactions within this group of teachers, to characterize the development of teachers' professional knowledge related to problem-solving in a dialogical process within a LS.

This theoretical and methodological paper first presents analytical tools from two different fields in education, namely mathematics education and dialogic analysis. Regarding the former, we introduce mathematical knowledge for teaching, particularly related to problem-solving; in relation to the latter, we use the models for the analysis of interactions. We then set out our research questions and describe the research's methodological elements: our use of qualitative and quantitative data analysis software. The paper ends with a discussion and outlines the perspectives of this research.

Mathematical Knowledge for Teaching Problem-Solving (MKTPS)

Much of the research on problem-solving has considered the student's point of view (for a survey on the state-of-the-art, see Liljedahl et al., 2016). A few authors have considered the teachers' point of view and the Mathematical Knowledge for Teaching (MKT) framework (Ball, 2017; Ball et al., 2008) to characterize the knowledge that teachers use to teach problem-solving. Wake and his colleagues (Foster et al., 2014; Wake et al., 2014) have attempted to broaden MKT to include mathematical process knowledge and pedagogical process knowledge, by rewriting the MKT categories to include 'concepts and processes' instead of just 'content' (Foster et al., 2014, p. 3.98). The focus on the process and not only on the knowledge is undoubtedly noteworthy. Nevertheless, during our data analysis, we realized that the specificity of knowledge for teaching problem-solving not only relates to the fact that this is a process. Similar insights were found in the work of Chapman (2005, 2012, 2015). In her conceptualization, based on a literature review from a sample of studies from 1922 to 2013, Chapman (2015) refers to MKT in order to distinguish between six categories of Mathematics Problem-Solving Knowledge for Teaching, divided into two groups: problem-solving content knowledge and pedagogical problem-solving knowledge (Table 1).

[1] For a presentation of lesson study, please refer to many IJLLS papers or, among others, to Lewis et al. (2019).

PS content knowledge	Knowledge of mathematical problems	Understanding of the nature of meaningful problems; structure and purpose of different types of problems; impact of problem characteristics on learners
	Knowledge of mathematical problem-solving (PS) *[do/meta]*	Being proficient in PS *[do]*. Understanding of mathematical PS as a way of thinking; PS models and the meaning and use of heuristics; how to interpret students' unusual solutions; implications of students' different approaches *[meta]*.
	Knowledge of problem posing	Understanding of problem posing before, during and after PS.
Pedagogical PS knowledge	Knowledge of students as mathematical problem solvers	Understanding what a student knows, can do and is disposed to do (e.g., students' difficulties with PS; characteristics of good problem solvers; students' PS thinking).
	Knowledge of instructional practices for PS	Understanding how and what it means to help students become better problem solvers (e.g., instructional techniques for heuristics/strategies, metacognition, use of technology and assessment of students' PS progress; when and how to intervene during students' PS).
	Knowledge of affective factors and beliefs *[teacher/student]*	Understanding the nature and impact of productive and unproductive affective factors and beliefs *[of the teachers/of the students]* on learning and teaching PS.

Table 1: Mathematics problem-solving knowledge for teaching (Chapman, 2015). The italicized parts are our contributions.

Moreover, quoting Mayer and Wittrock (2006, p. 287), she considers problem-solving as "a form of cognitive processing you engage in when faced with a problem and do not have an obvious method of solution". All these categories are influenced by the teacher's problem-solving proficiency and by his/her affective factors and beliefs. In line with Chapman's findings and for the purpose of bridging the mathematics problem-solving knowledge for teaching to the MKT categories, we propose the following graphical representation of this categorization (Figure 1).

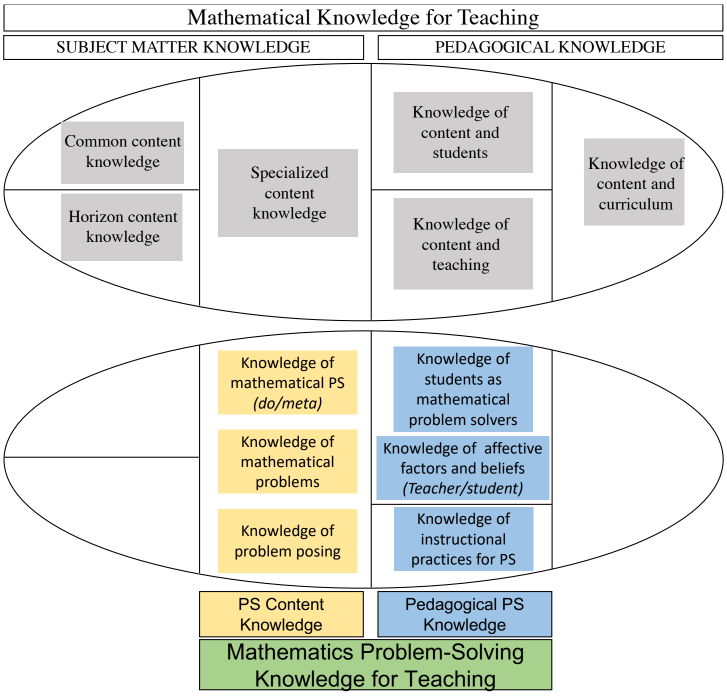

Figure 1: MKTPS. The upper grey-coloured part of the figure presenting MKT is from Ball et al. (2008); the coloured categories showing mathematics problem-solving knowledge for teaching are from Chapman (2015). The graphical representation of the coloured categories was carried out by the authors of this paper.

We then needed to determine knowledge levels (Table 2) to identify the participants' knowledge evolution throughout the meetings or the differences related to the roles (teacher, facilitator). These levels were derived inductively from our data. They do not constitute a hierarchy: contextualized knowledge or questioning are as valuable as generalized knowledge. The knowledge levels are labelled from 1 to 5.

Knowledge levels	
1	Inaccurate knowledge, lack of knowledge, self-assumed ignorance and/or questionable personal representation
2	Unexplained knowledge. Observation, testimony
3	Incomplete knowledge, knowledge with a low degree of certainty. Explicit questioning
4	Contextualized, explicit knowledge. Speaker knows or appears to know the rationale
5	Generalized, decontextualized knowledge (decontextualization process, possibly not fully completed). Generic example

Table 2: Knowledge levels

After presenting the nature of the knowledge, we examine the different ways in which it can be expressed and constructed over the course of the dialogue.

Interaction analysis

The origins of our approach

LS has been studied from various theoretical perspectives, particularly highlighting whether and how teachers' knowledge is developed. The research approach we present here focuses on analysing linguistic and interactional activities, as we consider this to be a privileged entry point from which to analyse the development of knowledge in this particular context. By favouring a descriptive and qualitative approach to the processes, we seek to highlight which conditions seem to favour the development of knowledge.

Many researchers are interested in the analysis of discourse and interactions in order to better understand the processes of knowledge acquisition. Without being exhaustive, let us mention in a French-speaking context, Balslev and Saada-Robert (2006); Filliettaz (2014); Mondada (2005) or in an Anglophone context, Kershner et al. (2020); Little (2002) and, particularly in the context of mathematics education and the framework of commognition, Cooper (2014); Kim et al. (2017); Sfard (2008). These works refer to different theoretical frameworks of educational sciences, linguistics, sociology or psychology. The term discourse or interaction analysis can hence refer to numerous approaches, which pursue various objectives or use different theoretical and methodological models. Each approach has its own characteristics in relation to the field of research and the study's objectives. However, they are all rooted in the work of Vygotsky and the socio-cultural model, and they recognize the importance of language and social interactions in the acquisition of knowledge.

In the context of our research, the work of an English research team, who developed a model called "interthinking" has especially met our needs.

Interthinking
This framework "appl[ies] the methods of Sociocultural Discourse Analysis" (Littleton & Mercer, 2013, p. 43). It aims to explain, mainly by using language, how "people are able to think creatively and productively together. We call this process 'interthinking' to emphasize that people do not use talk only to interact, they interthink" (p. 1). Based on the observation that when people work together, they are not always productive, "two heads are not always better than one" (p. 2), this group seeks to identify which kind of exchanges can be favourable for knowledge development.

In this approach, spoken language is, therefore, viewed as central, as it is directly linked to 'collective thinking', which is seen as a dynamic and creative process. While it is recognized that other forms of communication, such as images or non-verbal communication are also essential elements, the focus is on language exchanges, as they are seen as crucial to understanding how ideas and knowledge are co-constructed during a conversation. In this sense, Littleton and Mercer (2013) refer to the work of Bakhtin and explain that, when two people talk to one other, the meaning that each person infers from the conversation does not depend on a kind of "mental dictionary", according to which each person could link every word heard with a univocal definition. On the contrary, this process is not linear. Each statement is interpreted according to the context, to the knowledge of the subject and whether the interlocuter considers it necessary to understand what is being said. There are, therefore, different representations that confront one other, including misunderstandings, and it is precisely this dynamic that is a central element in the *interthinking* model. Indeed, the knowledge co-constructed between the conversation's partners is based on a context and a shared history, which helps to go beyond a superficial definition of words, so as to deepen their meaning and thus develop knowledge. By using language to express ideas and to agree on a shared meaning, the partners develop a new understanding of the topic being discussed, an understanding that neither of them would have achieved alone.

Therefore, the first element we retain in terms of research is that the process of knowledge development in interactions must be considered dynamic and creative. Consequently, questioning, challenging and confronting one other will be considered opportunities to develop knowledge, to deepen meanings and to collectively build a shared knowledge.

Following the analysis of recordings of students' speeches in classrooms, one of the first results of the *interthinking* group was to highlight three types of speech (Littleton & Mercer, 2013, pp. 15-16; Wegerif, 2020, pp. 28-29):

- *Disputational talk*: characterized by disagreements between the discussants and a competitive atmosphere in which each person makes their own decisions. There is little sharing of resources or constructive criticism, and interactions often relate to taking short positions without justification.
- *Cumulative talk*: characterized by speakers building positively but uncritically with regard to the comments of others. Everyone expresses their knowledge in the conversation, but there is no critical position or evaluation. Cumulative discourse is characterized by repetitions, confirmations and elaborations.
- *Exploratory talk*: characterized by everyone engaging critically but constructively in the conversation. Everyone brings information they consider relevant; this is questioned and argued, and group members seek to reach agreement before moving on to another stage. It is possible for an outside observer to follow the line of reasoning.

These types of discourse can be seen as different ways of thinking together. The third type was considered the most 'educationally effective' (Littleton & Mercer, 2013, p. 16). From a methodological point of view, the researchers state that these different types of discourse should not be considered a "coding scheme" or "categories of analysis" (Wegerif, 2020). They rather allow the identification of observational elements in order to understand the meaning of discourse in relation to knowledge development.

These findings have been widely replicated in a large body of research and professional development conducted by the Cambridge Educational Dialogue Research (CEDIR[2]) group. This research team aims to better understand and provide tools for developing dialogue and collaboration in the school and professional context. One initial inspiration was the work of Vermunt and his colleagues (Dudley, 2013; Vermunt et al., 2019; Vrikki et al., 2017; Warwick et al., 2016), who categorized dialogical processes in LS groups, in order to find statistical correlations between dialogical characteristics and teachers' professional development. As these categories are too broad for a detailed analysis, we

[2] See https://www.educ.cam.ac.uk/research/groups/cedir/

began to study the work of another CEDIR team, the Scheme for Educational Dialogue Analysis[3] group (SEDA, Hennessy et al., 2016; Vrikki et al., 2018), which forms the basis of the analysis grids we use in our method.

SEDA

This research group aims to develop a theoretical framework to systematically analyse classroom dialogues and highlight which dialogue structure can be observed in a lesson. They are particularly interested in dialogues in the classroom during problem-solving moments.

To this end, the SEDA group proposes to combine the *interthinking* framework, which we have just presented, with a *linguistic ethnography* approach (Hennessy, 2020; Hennessy et al., 2016). Referring to the work of Gee and Green (1998), Hennessy and her colleagues (2016) propose a framework for the coding and analysis of interactions, which takes up the principles of *the Ethnography of Communication*. This framework establishes three levels that make it possible to highlight the coherence related to the context in the analysis of a series of speech turns. The objective is to highlight the "meaning of the discourse" and to "form a coherent 'logic-of-inquiry' that recognises the importance of established educational process and cultural practices in shaping the meaning of teachers' and students' contributions" (Hennessy et al., 2016, p. 18). These levels of analysis are nested, and provide a systematic structure for the analysis of the data:

- *Communicative acts* are at micro-level and they are identified by their function in the interaction (asking a question, justifying, etc.); they usually correspond to an utterance produced by one person.
- *Communicative events* are at meso-level; they are defined by a series of speaking turns in which the participants, the modality (class, group, duo), the topic and the task remain constant.
- *Communicative situations* are at macro-level, and represent the context in which the conversation takes place (Hennessy, 2020, pp. 18-19).

The coding system, developed by the SEDA group, is more particularly situated at micro-level, as this group aims at characterizing the *communicative acts* in such a way as to highlight the dynamics of exchanges in the classroom. The authors specify that this coding system implies taking the temporal and chronological aspects into account. Indeed, researchers do not code an isolated statement, but a statement in the logic of a dialogue. During the coding process, there is a constant back and forth between the *communicative acts*, *communicative events* and *communicative situations,* which ensures that the analysis considers the overall logic of the dialogue.

Although we are operating in a different context, that of the LS, we share the same problems and objectives as the SEDA group, therefore, we have taken their grid and adapted it to our context, as explained below.

Lesson Study Dialogue Analysis (LSDA)

After having specified the research on which our interaction analysis methodology is based, we present our analysis grid in this section. In order to determine which type of dialogue would allow for the development of knowledge, we opted for a 'coding' approach (Hennessy, 2020) which involves systematically categorizing all interactions. In this way, we planned to highlight *patterns* that would be more 'productive' and to show the cohesive structure of the text (according to the *sociocultural discourse analysis* approach).

This led us to re-use the SEDA grid, which we adapted to our study context and called it Lesson Study Dialogue Analysis (LSDA). While the SEDA allows to characterize interactions between teachers and students, the LSDA focuses on interactions between trainers and teachers.

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[3] See https://www.educ.cam.ac.uk/research/programmes/analysingdialogue/

A long process of coding and discussion as a team was thus necessary to set up our grid of analysis, from our data and in an inductive movement. This required a certain degree of adaptation of the original grid, as we had to take into account our particular context, as well as that of the actors and their intentions.

The process of the modification of the SEDA scheme lies beyond the scope of this paper, but we will highlight two key modifications here, related to the type of exchange in a LS professional dialogue, which differs from a students' dialogue in a classroom situation. The first modification is related to the two SEDA categories, "B – built on ideas" and "R – make reasoning explicit", which were very similar. For example, in our data, it was almost impossible to distinguish between "B1 – build on /clarify others' contributions" and "R1 – explain or justify another's contribution". We, therefore, merged these two categories into "R – answer, develop". Since the question-response type of exchange among teachers was often present in our data, the codes for the R category were symmetrized with those of category Q – prompting development or reasoning (categories Q and R in Table 3), both entries being specified as clarification-justification-hypothesizing categories. The second modification is the adaptation of the "connect" category, due to the observation that group participants often make reference to incidents or episodes of the LS cycle or to other teaching experiences (category C in Table 3). This observation has already been illustrated in the analysis of the work of a different LS group (Ni Shuilleabhain & Clivaz, 2017) and we can relate it to the cumulative principle of Alexander (2018), "which underpins enquiry and knowledge growth in academic communities as well as classrooms, ensures that discussion is genuinely dialectical yet builds on what has gone before, advances understanding and is not merely circular" (p. 566).

Using the *Ethnography of Communication* levels of analysis, we systematically coded each turn of speech (identified as *communicative acts*) at a first level, in order to characterize the interactions and using one of the codes relating to one of the categories E, Q, R, P and G, as described in Table 3.
A second level highlights the area to which the *communicative events* refer. Our objective is to highlight the connections made in the course of exchanges. This level of coding is, therefore, carried out by blocks of several turns of speech, making it possible to highlight a form of sequencing of the interactions. A block corresponds to a sequence of interactions connected to the same reference: previous contributions, the research lesson, a teaching experience, a personal experience, a teaching representation, a reference, the LS process.

Finally, we have chosen to code the whole LS cycle. Each *communicative act* corresponds to one utterance, each *communicative event* to a series of utterances referring to the same topic (same code C, see Table 3) and the *communicative situations* correspond to the phases of the LS process: choice of topic; study of the topic, planning of the lesson, research lesson, analysis of the lesson.
The categories for LSDA are presented as follows in Table 3. The codes corresponding to each category are given in Appendix.

	Category	Features
Communicative acts (micro-level)	E – express or invite new ideas	This category marks the entry of a new subject into the discussion, a new idea, an observation. A distinction is made between invitations to formulate new ideas and the expression of a new idea.
	Q – arouse development or reasoning	This category is used with the next category, R, to code a series of exchanges around a subject. The Q-coded turn involves reference to a previous input. The three possible purposes of the Q-coded turn are to better understand a factual statement, to understand the reasons for a previous statement or to consider other possibilities or hypotheses.
	R – answer, develop	This category has three possible purposes: to provide clarification and explanation, to give a justification or to develop other possibilities or hypotheses.
	P – position or coordinate	This category is used to indicate a turn intended to mark one's stance or to coordinate ideas in relation to previous exchanges. It may involve synthesizing ideas, evaluating different perspectives, challenging an idea or taking a position or indicating approval.
	G – guide	This category is used to indicate a turn intended to guide the course of interaction by encouraging dialogue, by verbalizing the rules of communication in order to promote discourse, by proposing an immediate action, by proposing an action in the future, by taking an expert position or by focusing.
Comm. events (meso-level)	C – connect	This category is used to show the areas to which a series of exchanges refer, such as: • the content of a past discussion episode • the research lesson (past or future) • one's teaching experience • one's personal experience • the LS process (at meta-level) • beliefs about teaching and learning • the learning trajectory of participants • the mathematical task • a reference

Table 3: Categories of codes for Lesson Study Dialogue Analysis

Nesting the levels of communication

As stated above, the three levels of analysis are nested, and provide a systematic structure for the analysis of the data. Systematic coding is carried out at micro-level (communicative acts). Then, the aim is to characterize the communicative events in such a way as to highlight the "interaction function" in the sense of Dudley (2013). Therefore, each communicative event can be characterized as a type of discourse (arrow a→c in Figure 2). This type of discourse allows to analyse the LS phases (macro-level, arrow c→d in Figure 2).

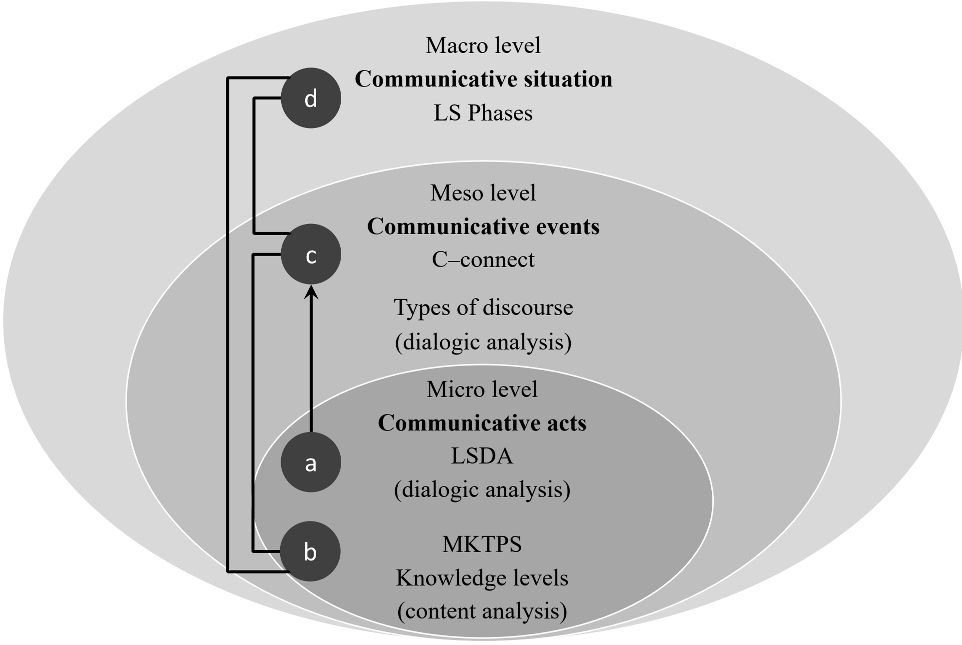

Figure 2: Nesting of analysis levels

Relationship between dialogue and knowledge: Research questions and method of data analysis

The analysis at micro-level deals with the succession of dialogic moves, the succession of MKTPS and the succession of knowledge levels (Clivaz et al., Submitted). However, the link between dialogue and knowledge cannot be made done at micro-level. Our ongoing analysis explores the possible relationship between the types of discourse at meso-level on the one hand, and the MKTPS and their levels at the micro-level (arrow c→b in Figure 2). This link between dialogic analysis and content analysis is at the centre of our attempt to understand the relationship between the dialogue and the construction of knowledge during a LS process (Daina et al., 2022).

In the case of our study, we needed to combine dialogic analysis (LSDA) with content analysis (MKTPS) to "get a multi-faceted insight into the empirical phenomenon in view" (Bikner-Ahsbahs & Prediger, 2010, p. 496). We refer to "combining" these frameworks in the sense of Prediger et al. (2008), since a combination does not require "the complementarity or even the complete coherence of the theoretical approaches in view" (Prediger et al., 2008, p. 173). In this case, the two frameworks are combined into a conceptual framework that is concerned with "second order variables (namely of relationships among the usual didactical variables)" (Arzarello & Bussi, 1998, cited by Prediger et al. (2008), p. 172). The conceptual framework captures two different points of view relating to teacher dialogue during a LS, as reflected in our research questions, heuristically structured around the main topics: problem-solving in mathematics and the LS process.

Research questions

Our research questions are summarized in Table 4.

Research questions and sub-questions	Level of analysis (Figure 2)
1. How is MKTPS built collectively during the LS process?	
1.1. What is the MKTPS that emerges during each LS meeting?	**d – b**
1.2. What are the transitions of two turns of speech, coded by MKTPS and those most represented per LS meeting and across all meetings?	**d – b**
1.3. How do instances of MKTPS follow one other in the dialogue?	**b**
1.4. What are the types of discourse in a LS group, related to the construction of MKTPS?	*c – d*
2. How do knowledge levels evolve?	
2.1. What are the knowledge levels used during each LS meeting?	**d – b**
2.2. What are the characteristics of the facilitators in terms of the levels of knowledge expressed?	**d – b**
2.3. Are the knowledge levels evolving differently depending on the type of discourse?	*c – b*
3. What are the dynamics of interactions of LS dialogue analysis at micro-level within the communicative acts?	**a**
4. What are the types of interactions during LS meetings that characterized the facilitators compared to those that characterized the teachers?	**a**

Table 4: Research question and levels of analysis.

The analysis related to these research questions and the results of the analysis are not found in this theoretical and methodological paper. They are presented in other papers (Clivaz et al., Submitted, for bold analysis in Table 4; Daina et al., 2022, for italic analysis in Table 4)

Method

To recapitulate, we analysed the work of a LS group of eight teachers and two facilitators in a LS cycle on problem-solving (five meetings of around 90 minutes and one research lesson). As our analyses are very detailed, we have embarked on a long-term project, and in the following, we will briefly illustrate certain elements of our method, that will allow us to sketch out the perspectives of our analysis and results.

In order to investigate these different questions, we use Transana, a qualitative data analysis program (Woods, 2002-2021). This software allows us to transcribe the dialogue of the five meetings and to code the interactions, according to the analysis grids we have just presented. Transana keeps a permanent link between the transcripts, the video recording and the coding (Figure 3).

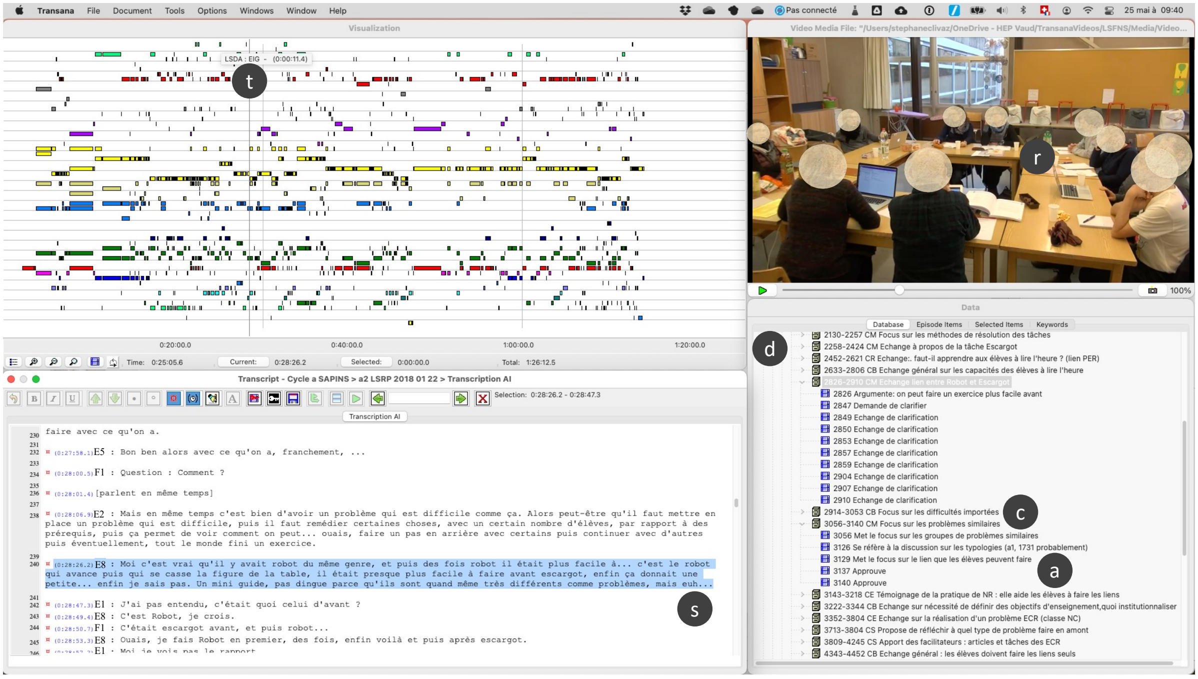

Figure 3: Transana window with interrelated video recording (r), transcript (s), codes' graphical representation (t), communicative acts (a) regrouped in communicative events (c) and communicative situations (d).

This is essential during the coding phase, which sometimes involves taking into account the tone of voice and the interaction's non-verbal context, so as to determine the code. Once the coding is complete, it is possible to carry out the first part of the analyses using Transana, making it possible to cross-reference and link these codes or create tables and diagrams that give an overall view of the coded data. It also highlights elements of descriptive analysis: speaking time per speaker, types of knowledge and levels, etc. All the transcripts and the analysis have been conducted in the medium French. An English translation is only performed for papers in English.

The passage from the coding phase to the analysis phase is facilitated by means of a quantitative process for the micro-level of analysis and a qualitative process for the links between the micro-level and the other levels. The quantitative process involves the export of the data from Transana to our own software and the creation of *Markov chains* (Gagniuc, 2017) in order to explore the temporal dependence of LSDA codes, MKTPS and levels of knowledge (Clivaz et al., Submitted). The qualitative analysis involves the recognition of the types of discourse (*exploratory*, *cumulative* or *disputational* talk, according to the *interthinking* model presented above) and based on the LSDA codes and on the interaction function of Dudley (2013). The links between these two types of analysis and our research questions are shown in Table 4.

Example of coding and analysis

We illustrate this coding with one extract only, due to space constraints. Table 5 shows the start of meeting 3, in which the participants discuss an article that they read between meeting 2 and meeting 3. Table 5 displays the communication events (meso-level, code C, see Table 3). At micro-level, the utterer, the time (minute and second), the summary of the utterance (the transcript is omitted), the LSDA code, the MKPTS and the level of knowledge is shown.

	Utterer	Time	Summary of the utterance	LSDA	MKTPS	Level
Connect to a reference	F2	0303	Provides the outline of the session and starts the discussion regarding the article	G		
		0325	A participant arrives			
	F2	0421	Invites thoughts relating to the article	E		
	T4	0441	Expresses that the appendix challenges what she is usually doing	E	K of instructional practices for PS	3
	T3	0622	Takes a stand: the article does not say what T4 does is wrong	P	K of instructional practices for PS	3
	T4	0641	Develops: different techniques are needed for different problems	R	K of instructional practices for PS K of math. problems	3 3
	T1	0657	Expresses importance of basic problems	E	K of students as math. problem solvers K of instructional practices for PS K of math. Problems	4 3 3
	T7	0747	Approves	P		
	T1	0749	Goes further on the importance of basic problems (continued)	R	K of math. Problems	4
	F2	0755	Approves	P		
	T1	0757	Goes further on the importance of basic problems (continued)	R	K of instructional practices for PS	3
Connect to teaching experience	T2	0841	Makes an observation about experimenting [name of a problem] in her class	E	K of students as math. problem solvers K of instructional practices for PS	2 2
	T6	0919	Approves	P	K of instructional practices for PS	3
	T2	0920	Provides clarification: certain students have been successful, but it takes too long	R	K of students as math. problem solvers K of instructional practices for PS	1 1
	T1	0936	Goes further on the usefulness of basic problems, particularly in the case of heterogeneous classes	R	K of students as math. problem solvers K of instructional practices for PS; K of math. problems	3 3 4
	F2	1019	Asks for clarification	Q	K of instructional practices for PS	3
	T1	1024	Justifies that her students are having difficulties	R	K of students as math. problem solvers	2
Connect to a reference	F1	1030	Expresses that it is not a basic problem	G	K of math. Problems	5
	T2	1032	Approves	P		
	F1	1034	Expresses that it is an atypical problem	G	K of math. Problems	5
	T1	1036	Goes further: students with difficulties can succeed when tackling basic problems	R	K of students as math. problem solvers	3
	T4	1057	Expresses an expert perspective: technique is not enough, representation is needed	E	K of students as math. problem solvers K of instructional practices for PS K of math. PS (meta)	3 3 4
	T6	1153	Goes further: balance must be found and drill must be avoided	R	K of students as math. problem solvers K of instructional practices for PS	3 3
	T1	1324	Asks if [name of another problem] is a basic problem	Q		
	T6	1327	Provides clarification: yes	R	K of math. Problems	1
	T5	1333	Takes a stand: no	P	K of math. problems	3
	T7	1334	Asks for clarification	Q		

Table 5: Coding of the beginning of meeting 3

From the coding of this extract, we can first highlight certain elements at micro-level for the two first communicative events (0303 to 1024). Facilitator F2 organizes the discussion at the beginning of the meeting (0303 and 0421, codes G and E). Then, participants take turns sharing their own experiences (0841 to 1024). The types of knowledge are mainly pedagogical problem-solving types of knowledge (see Table 1): knowledge of students as mathematical problem solvers and knowledge of instructional practices for problem-solving, at levels 2 and 3. Several new ideas are expressed (code E) but are not developed. At meso-level, this is one of the characteristics of a cumulative type of discourse and is found many times at the beginning of meetings.

Starting at 1030, the next communicative event is initiated by facilitator F1. Taking an expert perspective, he makes the link between the article and the teachers' experiences and questions. He then guides the discussion to the question relating to the difference between the types of problems. The expressed knowledge is at a generalized level (1030 and 1034). Starting at 1036 (and finishing at 2209, not in Table 5 due to space constraints) the following discussion takes place among the teachers. This consists of a succession of asking for justifications, as well as answers and development. This discussion makes the link between knowledge of mathematical problems on the one hand and knowledge of students as mathematical problem solvers and knowledge of the instructional practices for problem-solving on the other hand. These knowledge types alternate between explicit questioning (level 3) and contextualized explicit knowledge, in relation to which the speaker knows or appears to know the rationale (level 4). Moving from the LSDA codes to the meso-level, this communicative event is typical of an exploratory talk. The effect on knowledge can be seen in the link between problem-solving content knowledge and pedagogical problem-solving knowledge (see Figure 1) and in the back and forth between the level of questioning of knowledge and the level of justified contextualized knowledge.

This example of analysis shows some of the ways in which we can answer our research questions. As mentioned above, further coding and analysis are presented in other papers (Clivaz et al., Submitted; Daina et al., 2022) or are still to be conducted in order to answer the research questions in Table 4.

Conclusion

The development of teacher mathematical knowledge regarding the teaching of problem-solving in a collaborative setting is a complex process. Developing a framework to analyse this process has proven to be highly challenging for our team, and we consider this conceptual framework as an important result.

The framework allows to model, on the one hand, the knowledge relating to the teaching and learning of problem-solving and, on the other hand, the analysis of interactions during a LS. Our appropriation of the different theoretical models presented was conducted in relation to our research objectives and our data, using a process that was both deductive and inductive. This allowed us to operationalize our research questions and to anchor our method in a referenced theoretical framework.

From a methodological point of view, this theoretical anchoring gives us access to analytical grids and methods, which we have taken up in order to adapt them to our study context. We have shown in the article that, as a result of this work, our research has led to the development of tools that are specific to the analysis of interactions in a LS context.

From a theoretical point of view, our research constitutes an attempt to combine content analysis and dialogic analysis, and to find ways of linking the micro-level of analysis with the meso-level. We hope that the dialogue that we have initiated with the researchers from the CEDIR group on the one hand and with our mathematics education colleagues on the other, will contribute to developing research examining the way in which teachers' knowledge is constructed in the dialogue, particularly in the context of LS.

Our analysis involves a "systematic coding" approach (all turns of speech are coded) which is extremely time-consuming. However, it allows for a very detailed analysis of the interactions in relation to the evolution of knowledge. Moreover, once the data have been coded, this systematic approach offers the possibility of questioning the coded data in various ways, which brings a great deal of richness to the analyses.

Beyond the research questions presented here, the data developed in conjunction with our grids can indeed answer many other questions related to the LS process, particularly regarding the role of the facilitator or the effects of his/her interventions on the evolution of knowledge in a LS. Our forthcoming results should also contribute to teacher educator training, by highlighting which types of intervention are favourable to the development of knowledge. These results could also lead to the development of tools that could accompany facilitators during LS. Inspired by the work of the SEDA team, which has developed tools for teachers to promote dialogue in the classroom, based on their research, we hope to be able to provide tools that could be used in facilitator training, such as those developed by the Lausanne Laboratory Lesson Study[4].

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[4] See www.hepl.ch/foffa

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Appendix: LSDA codes

Category	Code	Definition
E – express or invite new ideas	EI	Invite to express ideas
	EE	Express an idea, make an observation
Q – arouse development or reasoning	QC	Ask for clarification
	QJ	Ask for justification
	QH	Ask to consider a hypothesis in relation to the above
R – answer, develop	RC	Provide clarification
	RJ	Give a justification, go further, develop
	RH	Develop a hypothesis
P – position or coordinate	PS	Synthesise ideas
	PE	Assess different perspectives
	PC	Propose a consensus
	PV	Challenge an idea, a point of view
	PP	Take a stand
	PA	Approve, disapprove
G – guide	GG	Encourage dialogue and dynamics within the group
	GA	Propose an action, a questioning
	GE	Express an expert or authoritative perspective
	GF	Focus on
	GP	Propose, project an action into the future
	GD	Discuss about the speech
Humor	H	Humor
C – connect	CA	Refer explicitly to previous contributions
	CL	Refer to the research lesson
	CE	Refer to teaching experience, talk about teaching practice in general
	CH	Refer to personal experience
	CR	Refer to a reference, say that a reference is missing
	CS	Reflect on, take into account the LS process
	CB	Refer to a belief, a representation of teaching,
	CT	Make explicit the achievements, shortcomings and trajectories of the participants in relation to the LS
	CM	Carry out a mathematical and didactic analysis of a task or a sequence. Do the task