TOWARDS A MICRO ANALYSIS OF TEACHERS' INTERACTIONS IN A MATHEMATICS LESSON STUDY GROUP

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This contribution presents an ongoing study of dialogic interactions in a lesson study group about the teaching of problem-solving in mathematics. The paper focuses mainly on the construction of the grid of analysis, its roots in previous research and its potentiality to analyse the specific dialogic role of each facilitator and teacher during the phases of lesson study process.

Introduction

The theme of collaboration in teacher professional development is an increasingly important theme, particularly in mathematics education as shown in the ICME-13 Survey (Jaworski et al., 2017) and the future ICMI-25 study. Among the implied conditions for collaboration, effective communication and promotion of professional dialogue (Boavida & Ponte, 2002) are put forward in ICMI-25 study discussion document. The study of communication among teachers, more precisely on how teachers learn through interaction with others, questions particularly the nature of interactions between the facilitator and the teachers (van Es, Tunney, Goldsmith, & Seago, 2014). The goal of our research is to describe these dialogic interactions in order to better understand how teacher learning occurs in a lesson study process. By listening to the voice of every teacher and facilitator and by characterising their interaction, the ongoing micro analysis, conducted as a comprehensive case study, is a rather new direction of research. It would also make a significant contribution to the topic of ICMI-25 study, "Roles, identities and interactions of various participants in mathematics teacher collaboration", specifically to address the two questions:

- How are different roles and identities shaped and developed among various "actors" (teachers, leaders, mathematicians, researchers, etc.) within a collaborative group? How do lead teachers negotiate their dual roles and identities as both teachers and facilitators of peer-collaboration?
- How can different stakeholders impact teacher collaboration?

Our research is still in its initial stage and this paper concentrates mainly on its theoretical and methodological aspects. After presenting a brief background of lesson study, the context of our study and our research interests, the article focuses on the construction of our grid of analysis and, in link with the "Theoretical perspectives on studying mathematics teacher collaboration" theme of ICMI-25, its roots in previous research. The formulation of our research questions and the perspective for the research concludes this article.

Lesson Study

Jugyo Kenkyu, literally lesson study (LS), were born in Japan in the 1890s. They were popularised in the 2000s following international TIMSS comparisons (TIMSS & PIRLS International Study Center, n.d.; TIMSS Video, n.d.) and the comparison between mathematics education in Japan, Germany and the USA that Stigler and Hiebert (1999) presented in *The Teaching Gap*. Thanks to the efforts made to promote LS, and in particular to the work of Lewis, who contributed to formalising and popularising LS in the USA (C. Lewis, 2002, 2015; C. Lewis & Hurd, 2011), LS was initially introduced in the USA as a professional development approach to improve US mathematical classroom teaching and learning (Yoshida, 2012). As a mode of professional development, LS has developed all over the world leading to an establishment of the World Association of Lesson Studies (World Association of Lesson Studies, n.d.) and it has attracted the interest of many researchers in educational sciences, particularly in mathematics education (see, for e. g. among the most recent international edited books, Huang, Takahashi, & da Ponte, 2019; Quaresma et al., 2018).

LS starts from an area of difficulty in teaching and learning identified by a group of teachers. Teachers analyse the targeted learning, study the mathematical concept, consult the various teaching methods, study articles from professional journals and other resources. This study allows them to plan a lesson together. This lesson is implemented in the classroom of one of the group members. Other teachers observe the lesson in real-time and analyse its impact on students' learning. The group may decide to plan an improved version of the lesson to be conducted in another teacher's classroom and the loop begins again. The results of the work are disseminated, both in the form of a detailed lesson plan for future use by other teachers and also in the form of articles in professional journals.

LS groups are usually led by an experienced teacher or trainer, called a facilitator, who "keeps the conversation moving and fair. Involves all participants. Follows an agreed-upon agenda" (C. Lewis & Hurd, 2011, p. 124). While in Japan, LS are sometimes facilitated directly by the group members, they almost always involve one knowledgeable other who provide feedback in the discussion after the research lesson and sometimes another knowledgeable other who can draw attention to key elements during the planning phase (Watanabe & Wang-Iverson, 2005). In countries where LS are developed (particularly in Japan) the role of facilitators as facilitators participating in the group and that of occasional external experts is very well defined, these two roles are often assumed by the same person or are confused in places where LS are starting to take root (Clivaz & Takahashi, 2018).

Elements of Context and Research Interests

Our research is part of the work of the Lausanne Laboratory Lesson Study (3LS, n.d.) which studies the work of several LS groups in the French-speaking part of Switzerland. We are currently analysing the work of a LS group composed of 8 grade 3 and 4 teachers from the Lausanne region and two facilitators. The two facilitators consist of a mathematics educator (the first author of this contribution) and a teacher from the institution who participated as a member of a previous LS group in mathematics. From 2018 to 2019, this group has completed three LS cycles with the question: “how to teach grade 3-4 students how to solve mathematical problems”. The fact that the teachers will be in a (professional) problem-solving situation about problem solving teaching is coherent with Ball and Cohen (1999) suggestion that professional development programs should situate teacher learning in the types of practice they wish to encourage. The data from these two cycles were analysed, using Transana (Woods, 2002-2017) to encode video recordings which are integrated with the transcripts. The first cycle mentioned in this article, includes eight meetings that lasted for about 90 minutes each and two research lessons. Interviews with the two facilitators were also analysed as a way of data triangulation. Since other researches of our team used the same type of data (e.g.

Batteau, 2017; Clivaz & Ni Shuilleabhain, 2019), the obtained results show that this type of data is thorough, systematic, reliable and authentic regarding the perspectives and practices of participants.

The research interests of our team [1] are two-fold. The first series of questions is about the mathematical knowledge related to problem solving that teachers use during this LS, the second series aims to describe the types of interactions related to the construction of this mathematical knowledge and, ultimately, to describe how the mathematical knowledge is constructed in a LS group. This paper will concentrate on the analysis of the interactions and only show the links with the mathematical knowledge and the problem-solving-teaching. In order to do so, we will present some of the previous studies and the analytical frameworks which is currently developed by our group.

Previous Research

It is with the objective of accurately describing how teachers' knowledge is constructed or evolves and, more generally, to better understand what happens between actors within an LS process, that we have been led to focus on discourse analysis in a sociocultural perspective, rooted in the work of Vygotsky (1962, 1978) for whom the acquisition and use of language transforms children's thinking. One of our first inspirations was driven from the work of Vermunt and his colleagues (Vermunt, Vrikki, van Halem, Warwick, & Mercer, 2019; Vrikki, Warwick, Vermunt, Mercer, & Van Halem, 2017; Warwick, Vrikki, Vermunt, Mercer, & van Halem, 2016) who categorise the dialogic processes in LS groups in order to find statistic correlations between certain dialogic features and teachers' meaning-oriented learning in LS. These categories being too broad for a comprehensive analysis we were led to study the work of a sister group within the Cambridge Educational Dialogue Research group (CEDiR, n.d.), the Scheme for Educational Dialogue Analysis (SEDA, Hennessy et al., 2016) group. Rooted in the work of Alexander (2008) about dialogic teaching, this group produced a comprehensive grid to analyse classroom dialogue in problem solving situations. The grid and the method of an "inductive-deductive cycle that allowed to distil out the essence of dialogic interactions and operationalise them in the form of a new systematic indicators for these productive forms of educational dialogue" (Hennessy et al., 2016, p. 17) seemed to be a good choice to serve as a basis for the construction of a grid of systematic indicators able to capture the forms of professional dialogue within a professional development process. Nevertheless, adaptations have to be made to SEDA scheme to take into account our research context as well as previous research on teacher learning in LS.

Comprehensive research on LS groups and the fact that they appear to have an impact on teachers' professional knowledge often focuses on the essential role of facilitators (e.g. Akiba, Murata, Howard, & Wilkinson, 2019; Bjuland & Helgevold, 2018; Borko, Koellner, & Jacobs, 2014; Carlson, Moore, Bowling, & Ortiz, 2007; Hart & Carriere, 2011; C. Lewis & Hurd, 2011; J. M. Lewis, 2016; Schipper, Goei, de Vries, & van Veen, 2017; Stepanek, Appel, Leong, Turner Mangan, & Mitchell, 2007) and possible knowledgeable other (e.g. Amador & Weiland, 2015; Akihiko Takahashi, 2014; A. Takahashi & McDougal, 2018; Watanabe & Wang-Iverson, 2005). While many studies mention the importance of these roles and give examples of facilitator interventions or mention statements by teachers saying how important this role is to them, qualitative studies describing precisely how this

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role allows teachers to build professional knowledge are rare to date. While surveys such as Akiba et al. (2019) show that "facilitators' focus on student thinking, the quality of materials, and duration of lesson study were significantly associated with teacher participation in an effective inquiry process, which in turn is associated with perceived positive changes in teacher knowledge, self-efficacy, and expectation (p. 352)", these studies do not address precisely what makes facilitator interventions facilitate the construction of professional knowledge and this group calls on further exploration “by analysing the detailed nature of interactions between facilitators and teachers in each stage of lesson study. (p. 362)”.

For our part, in our previous research, we examined the evolution of the trainer's role in terms of knowledge sharing in a series of LS (Clerc-Georgy & Clivaz, 2016). Apart from that, we have also theorised the work of LS groups in relation with French *didactique des mathématiques* (Clivaz, 2015, 2018). We also showed which Mathematical Knowledge for Teaching (MKT, in the sense of Ball, Thames, & Phelps, 2008) teachers use during the LS process (Clivaz & Ni Shuilleabhain, 2019; Ni Shuilleabhain & Clivaz, 2017). Nevertheless, interactions within the group in particular between the facilitators and the teachers are yet to be explored.

Methodology for Analysing the Types of Interactions

In this section, we will describe our grid for analysing interactions as the result of a process that is both deductive and inductive. This grid currently focuses mainly on enunciative modalities but will be linked in the rest of our research with MKT and issues related to the topic of problem solving.

The Construction of the LS Interaction Analysis Grid

Composed of 33 codes grouped into 7 entries, the SEDA grid (Hennessy et al., 2016; SEDA, n.d.) had to be adapted from students’ mathematics problem solving classroom situation to teachers’ professional problem solving discussion. We started from the SEDA grid, using the same codes every time it was possible and adapting them when necessary. After a one-year coding work, and team discussion of the coding, we were able to set up, in an inductive way, our grid for analysing the interactions within a LS. This required a fairly radical adaptation of the original grid, as we had to take into account our particular context as well as the actors and their intentions.

The process of the modification of the SEDA scheme lies beyond the scope of this paper, but we will highlight here the two main modifications related to the type of exchange in a LS professional dialogue which differs from a students’ dialogue in a classroom situation. The first characteristic is the Question-Response type of exchange among teachers that was often present in our data. This led us to reorganise the SEDA entries related to “Invite elaboration or reasoning”, “Make reasoning explicit” and “Build on ideas” into “Arouse development or reasoning” and “Answer”, both entries being specified into clarification-justification-hypothesising categories. The second modification is due to the observation that group participants often make reference to incidents or episodes of the LS cycle or to other teaching experience. This observation has already been illustrated in the analysis of the work of a different group (Ni Shuilleabhain & Clivaz, 2017) and we can relate it to the cumulative principle of Alexander (2018) “which underpins enquiry and knowledge growth in academic communities as well as classrooms, ensures that discussion is genuinely dialectical yet builds on what has gone before, advances understanding and is not merely circular” (p. 566).

Presentation of the Analysis Grid

The first unit of coding of our analysis is the conversational turn, which allows us to first code the identity of the speaker. Like the SEDA grid, our LS Dialogue Analysis (LSDA) grid is organised into 6 entries that group 37 codes that allow us to characterise interactions within the LS. Each code is characterised by indicators, assuring a good validity of the coding (interrater reliability tests will be conducted in Fall 2019). We have distinguished two levels of coding (see Table 1):

- Categories E, Q, R, P and G allow us to highlight a dynamic of talk. Each conversational turn is specifically coded. In some cases, a turn involves several types of utterances, in this case, it is cut out in order to have a more precise coding.
- Category C allow us to show what is being used as a reference in the conversational turn. This enables us to be aware of the connections that are made during the exchanges. In this case several turns are coded as a group.

The categories for LSDA are presented as follows in Table 1.

Table 1: Categories of codes for LS Dialogue Analysis

Category	Features
E – Express or invite new ideas	This category marks the entry of a new subject into the discussion, a new idea, an observation. Distinction between invitations to formulate new ideas and expression of a new idea is made.
Q – Arouse development or reasoning	This category is used with the next category R to code a series of exchanges around a subject. The Q-coded turn involves reference to a previous input. The three possible purposes of the Q-coded turn are, to better understand a factual statement or to understand the reasons for a previous statement or to consider other possibilities or hypotheses.
R – Answer	This category is used with the previous category Q in an exchange. The three possible purposes of the R-coded turns are, to provide clarification and explanation or to give a justification or to develop other possibilities or hypotheses.
P – Position or coordinate	This category is used to indicate a turn intended to mark one's stance or to coordinate ideas in relation to previous exchanges. It may involve synthesising ideas, evaluating different perspectives, challenging an idea or taking a position, approving, recognising a change in position.
G – Guide	This category is used to indicate a turn intended to guide the course of interaction by encouraging dialogue, by verbalising the rules of communication in order to promote discourse, by proposing an immediate action, by proposing an action in the future, by taking as an expert position, by providing feedback, by focusing.
C – Connect	This category is used to show what a series of exchanges refers to. It might refer to: • the content of a past episode of the discussion • the research lesson (past or future) • one's teaching experience

	• one's personal experience • a theory or to mathematical principles • the LS process (at a meta level)

Our Research Questions

The ongoing coding of our transcribed LS meetings allow us to have three levels of coding:

1) Identity of the speaker
2) Dynamic of talk (categories E, Q, R, P, G)
3) The topic to which the series of utterances are connected (category C)

These three levels and the relationships between them allow to operationalise our question about the interactions during LS. The description of the interactions in terms of dynamic of talk (2) will show the type of exchanges that can take place within a LS, answering to the questions: is the pattern in dialogue E then Q/R the most frequent? when do P and G take place?

The type of intervention (2) and the changes of reference (3) linked to the identity of the speaker (1) will allow us to determine the specific dialogic role of each facilitator and of each teacher during the phases of the LS process, answering to the questions: are the facilitators mainly in charge of making the discussion progress or is it a shared responsibility? are these roles evolving along the process?

Conclusion and Perspectives

This research is still in its initial phase and the analyses will be conducted, using the LSDA categories, as described above. Detailed analysis will be conducted for key moments of the three LS cycles about the teaching of problem solving conducted by this group for two years. A second series of analysis will be conducted about MKT in the direction of research which is in the similar vein with Clivaz and Ni Shuilleabhain (2019). The most important part will then be to linked these two parts in order to describe how the mathematical knowledge is constructed within a professional dialogue in a LS group and what the role of a facilitator is, in the construction of this knowledge. The fact that one of the two facilitators of the group is a teacher constitutes a way to not only listen to the voice of the teachers in their dialogue, but also to characterise their voice when they contribute to facilitating their own learning.

The facilitator's facet of this research is expected to give fruit in at least four aspects. The first is to contribute to the series of research of the Réseau Education et Formation group about "the role of the trainer in mathematics education programs". The second is to follow up of the research interest about MKT for primary teachers (Clivaz, 2014, 2017) with a LS contextualised exploration of Mathematical Knowledge for Teaching Teachers (Zopf, 2010) or of Mathematical Knowledge for Professional Development (Borko et al., 2014). The third and the fourth aspect are headed in the direction of the training of facilitators with a chapter in a collective book for LS facilitators (Murata & Lee, in preparation) and by the organisation of a summer school for training French-speaking LS (and other collaborative professional development) facilitators in Lausanne in July 2020. But our main hope is to participate in ICMI-25 study through the sharing of our preliminary results of this analysis and with the tools we developed, in order to analyse collaborative professional discussion of teachers working and learning in collaborative groups.

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