Cultural effects on mathematics lessons: through the international collaborative development of a lesson in two countries

Stéphane Clivaz[1] and Takeshi Miyakawa[2]

[1]Lausanne University of Teacher Education, stephane.clivaz@hepl.ch

[2]Joetsu University of Education, miyakawa@juen.ac.jp

This paper analyses two grade 4 mathematics lessons given in Switzerland and in Japan by student teachers (pre-service teachers) in the context of a project-based international exchange program. The lesson, initially planned together by the nine student teachers of the two countries, was finally realised in quite different ways in Switzerland and in Japan. Using the notion of levels of didactical codetermination, the analysis makes explicit the differences of the two lessons and identify cultural elements that shape such lessons.

Keywords: Comparative education, lesson study, levels of codetermination, cultural context.

Introduction

International comparative studies have been carried out so far on different aspects of mathematics education. Large-scale studies such as PISA, TIMSS, and TALIS provide extensive information on the education, while small-scale studies carried out by individual researchers complement the large-scale studies and allow understanding in-depth on the specific aspect (Cai, Mok, Reddy, & Stacey, 2016). We are interested in the small-scale comparative studies on the mathematics lessons. Previous comparative studies on mathematics teaching have shown the large differences between Asian and other countries (Cai & Wang, 2010; Clarke, Emanuelsson, Jablonka, & Mok, 2006; Clarke et al., 2007; Stigler & Hiebert, 1999; Stigler & Perry, 1988). One of the complexities of classroom comparative studies is to have comparable data. Mathematics teaching one may observe in the classroom may vary from one lesson to another under the effects of so many different factors inside and outside classroom.

We recently obtained data in the context of an international exchange of student teachers and their educators. The student teachers from Switzerland and Japan prepared a lesson together and realised it separately in each country. As the lesson is developed collaboratively, the differences we may identify in the implemented lessons would be deeply rooted in the educational culture of each country. We think that such data allows an interesting comparative study on mathematics lessons. The aim of our study is thus to advance understanding of mathematics teaching and learning of different countries, what are the characteristics and what elements shape such characteristics, through an analysis of the data collected in the project-based international exchange program.

Theoretical frameworks

One issue on the methodology of international comparative studies of mathematics classroom is to set up a common criterion to analyse the data collected from two countries. It is not necessarily easy to decide what to compare, because it would vary according to how we characterise the classroom activities and what we consider important with mathematics lessons. The lesson structure is one of aspects which has been compared in the previous studies (Clarke et al., 2007; Stigler & Hiebert,

1999). In such studies, it is important to find out a lesson structure of reference which includes significant phases in terms of mathematics learning.

In our study, we draw attention to the lesson called *mondai kaiketsugata jugyō* in Japan (structured problem solving lesson in English; see Stigler & Hiebert, 1999) and to *the theory of didactical situations* (TDS hereafter; Brousseau, 1997). The former lesson usually consists of four or five phases: introduction of a problem, individual work and/or group work, *neriage* (whole class collective work), and *matome* (synthesis) (see also Shimizu, 1999). The latter characterises the process of mathematics teaching and learning, in terms of the states of mathematical knowledge—*situation of action*, *situation of formulation*, and *situation of validation*—as well as the process the teacher concerns—*devolution* and *institutionalisation*. For the analysis of lessons in our comparative study, we adopt the characterisations of these previous studies with some adaptations in order to take into account the specificities of Japanese lesson as well as the different states of mathematical knowledge during learning.

While these frameworks provide us with the aspects of classroom activities to be compared, they do not allow us to characterise the exterior factors that shape the lesson. To deal with this issue, we rely on *the anthropological theory of the didactic* (ATD hereafter) which directs us to investigate the factors beyond the classroom (Bosch & Gascón, 2006; Chevallard, 2002). In this theory, the lesson implemented in the classroom is considered as a result of *didactic transposition* which is under the influences of the *conditions* that support the realisation of such lesson and the *constraints* that hinder it. ATD implies that these conditions and constraints may have the different nature beyond those identifiable in the classroom, and proposes a classification called *the levels of didactic codetermination*: *civilisation – society – school – pedagogy – discipline – domain – sector – theme – subject* (Bosch & Gascón, 2006; Chevallard, 2002). The study by Artigue & Winsløw (2010) shows that this perspective allows us to capture, in the context of international comparative study, the extensive factors that affects mathematics education. We also consider that such perspective helps us to identify different cultural effects that shape the lessons of our project.

Context of this study

Our principal methodology of comparative studies is based on the collaborative development of a mathematics lesson by Swiss and Japanese student teachers. This happened in the more general context of students and professors exchange program, called PEERS (Projet d'Étudiants et d'Enseignants-chercheurs en Réseaux Sociaux, Student and Researcher Social Networks Project) carried out by Lausanne University of Teacher Education (HEP Vaud). This project articulated student exchanges around a jointly defined research project by a group of students from the HEP Vaud in association with a group of students from the partner university. Each PEERS is supervised by a teacher-researcher of each institution, combining face-to-face (one week in fall and another week in spring) with distance collaborative work phases. PEERS with Joetsu University of Education was supervised by the two authors of this paper.

The group first met through Skype meetings organised three times in fall 2017, and decided the general theme of PEERS and the mathematical theme: the collaborative development of a problem solving geometry lesson for grade 4 pupils, like the lesson study process (Hart, Alston, & Murata,

2011). The group spent one week in Joetsu in October 2017 for designing a task, studying the topic and planning the lesson together. At the end of this week, a first draft of lesson plan was ready.

During the winter, the two groups developed their lesson separately and taught them several times. For the Japanese group, the lesson was taught two times as a mock lesson and two times in grade 4 classes with about 35 pupils in the attached school. For the Swiss group, the lesson was taught by each Swiss student in her/his practicum classroom of about 20 pupils with the observation by the rest of the group, and followed by a post lesson discussion. This discussion led to changes in the lesson plan for the next lesson. After three Skype meetings, the Japanese group spent one week in Lausanne in February 2018. During this week, the group observed the last Swiss lesson, watched the video of the last Japanese lesson, and discussed the differences and commonalities.

The problem the group selected was the one in the Swiss textbook (Danalet, Dumas, Studer, & Villars-Kneubühler, 1999). The question is: "Divide a square into several squares, but not more than 20. Find as many solutions as possible". The lesson plan by the Swiss students is available on the websites of Lausanne Laboratory Lesson Study (www.hepl.ch/3LS).

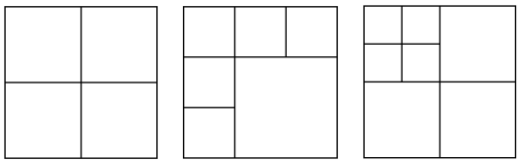

Figure 1: Some of the possible solutions for 4, 6 and 7

Methodology: data collection and analytical tools

The data were collected from the above-mentioned exchange project. We videotaped most of the activities related to the collaborative development of mathematics lesson and its implementation: Skype meetings, discussions in the face-to-face meetings, preparatory lessons, implementation of lessons, post lesson discussion, etc. In this paper, we principally analyse, for the comparative study, the last versions of lesson plan and the video data of the last lesson from the two countries. The first draft of lesson plan was collaboratively written in English. Then, the detailed lesson plans were written separately in both sides in students' own languages (Japanese and French) and revised several times after the lessons. They were translated later in English for sharing in the project. The Japanese video data was transcribed first and then translated into English, while the Swiss data was transcribed in French and then only the parts necessary for writing this article was translated in English, because both authors understand French.

For the comparison of lessons, we characterise the process and structure of lesson by identifying the modes of students' and teacher's works in the classroom from two aspects of teaching and learning activities. The first is the interaction among pupils and teacher that implies three kinds of works: individual work, group work, and collective or whole class work. This aspect allows us to describe the overall activities in the classroom as well as the roles played by teacher and pupils. The second aspect is the mode of working in terms of the process or phases of problem solving: introduction, research, sharing, and synthesis. In this way, we are able to capture the structure of the lesson with the description of overall classroom activities. These two aspects are consistent with the structure of Japanese problem solving lesson.

Further, we investigate the characteristics of activities throughout different phases of the lesson. Adopting the viewpoint of TDS that characterises the evolution of mathematical knowledge, we analyse the devolution process—how the responsibility on the given task moves to the pupils—, the validation—how the teacher validates pupil's answer; how the teacher makes pupils find the validity of their answer—, and the institutionalisation process—what kind of knowledge is institutionalised as an object to be learnt.

While the student teachers designed a single task together, their implementation should be under the several implicit constraints of each country, and it is expected that we may identify several differences between the implemented lessons, due to the factors which are deeply rooted in the teaching culture they are belonging to. We try to identify these factors according to the levels of didactic codetermination, by focusing on the differences identified in the comparative analysis and by exploiting all available resources at our disposition.

Comparative study of Swiss and Japanese lessons

Even though the task was designed collaboratively in the face-to-face workshops organised in Japan, its implementations in Switzerland and Japan were very different. We found the differences between the two countries in different phases, both between the structures of the lessons (see Figure 2) and between each of these parts. In what follows, we focus on the differences of the validation during the research phase and the sharing phase.

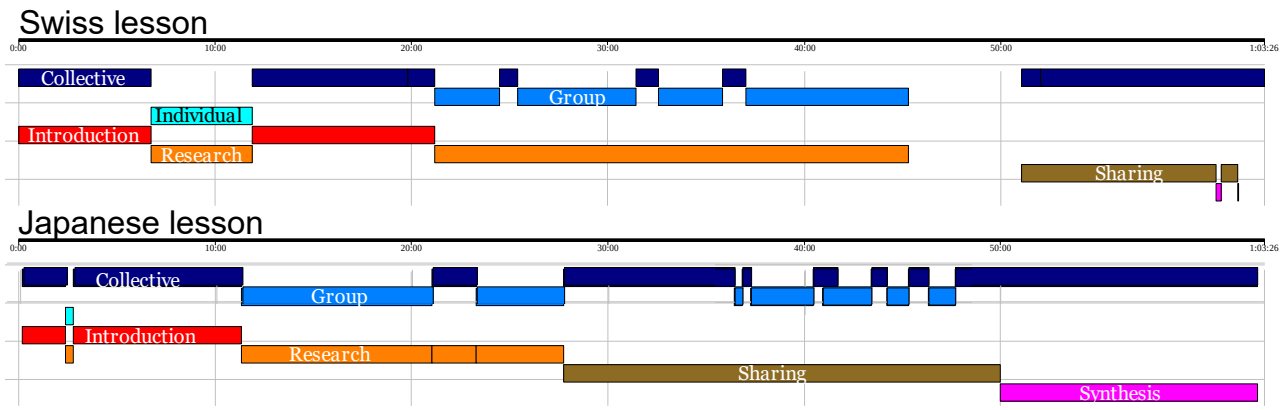

Figure 2: Structure of the two lessons

The issue of validation is at the heart of mathematics (Balacheff, 1987; Lakatos, 1976) and it was the principal and recurring difficulties for Swiss students as well as Japanese students when designing, teaching and discussing the lesson. When finalising the planification in their own languages, Japanese and Swiss students are not specific about validation. Japanese lesson plan says:

> When an incorrect answer is given, the teacher takes it up to the whole class when necessary and checks why it is wrong.

The Swiss lesson plan is not more precise about the criteria for validity, but it is more specific about a list of incorrect solutions.

> Show on the board correct and [...] incorrect solutions (diagonal, cut in half). Define the criteria for a correct solution together with the pupils, write them down on the blackboard to make a check-list

[of incorrect solutions] to which the pupils will have to refer before coming to show a solution to the teacher.

This list reflects the main preoccupation of the Swiss team to deal with many pupils coming to the teacher during the group research phase (in orange in Figure 2) to ask him/her: “is this correct?”. In fact, during the research phase, the Swiss teacher takes care of pupils one by one in front of the board and tells if the solution is correct or not:

Pupil: Teacher, is it okay?

Teacher: Ah, a box inside the square. […] unfortunately, [first name], your solution, I cannot accept it, because squares in the square, in the square ...

In comparison, the Japanese teacher moves from one group to another and asks questions:

Teacher: This one, are they really all squares? Could you think about it?

Pupil: Okay. [Teacher leaving]

In fact, the way of validation of the solutions by the Japanese team is close to the definition of validation by Margolinas (2004): “the pupil decides by himself about the validity of his work [...] thank to the interactions with the *milieu* (p. 24)”. In contrast, the Swiss team is making an evaluation: “the validity of the pupil’s work is evaluated by the teacher in the form of an irrevocable judgement (p. 24)”.

This characteristic can also be found in the sharing phase (in bronze colour in Figure 2), usually called *mise en commun* (putting in common) in French and *neriage* in Japanese. This can be seen in the transcription of the lessons and the student teachers are aware of these differences. After observing together the Swiss lesson and watching the video of the Japanese lesson, they write in English in their collective reflections:

Validation of answers during “neriage”: JP = others students / CH = the teacher mostly.
In Japan [it] is very important to exercise the students/children to think about HOW they find a solution. By explaining from peers to peers (and not the adult explaining), other students will understand it more because it comes from another student like them. It also helps the students to confirm that he understood well the problem and it’s induce a discussion and deep-thinking on the topic. (Notes of workshop, PEERS week in Lausanne)

The student teachers’ sharp and concise description of the difference between who is validating the solutions in the two lessons comes quickly to the search of more general reasons. The differences concerning the validation between Japanese and Swiss lessons here could be summarised in the differences of two aspects of mathematics lesson which are mutually related. The first aspect is the overall form of mathematics teaching: *collective teaching* and *individualistic teaching*. In Japanese lesson, the *neriage* is a moment for the whole class, including pupils and teacher (actually it was the teacher who manages this phase), to validate pupil’s answer and further develop their ideas, and even in other phases, the teacher often tries to control the whole class (see the duration of collective work of Japanese lesson in Figure 2), while in the Swiss lesson the teacher individually validates pupils’ answers in both research and sharing phases. The second aspect is the *didactical contract* (Brousseau,

1997) that determines what can be done by pupils and teacher: the teacher may directly validate pupils' answers in the Swiss lesson, while not in the Japanese lesson; the pupils may ask the teacher to validate their answer in the Swiss lesson, while not in the Japanese lesson. In the actual lesson, the Japanese teacher might not always play well her role as she was still a student, but she was trying to leave the responsibility of validation to the individual pupils or group in the research phase and to the whole class in the *neriage* phase. The question we ask is: what makes such differences? We investigate and discuss the cultural effects on these two aspects in the next section.

Cultural effects on mathematics lessons

Collective teaching or individualistic teaching

One obvious factor that supports, or even requires to carry out, the collective teaching in Japanese class is the number of pupils. In the classroom of 35 pupils (about 20 pupils in Swiss case), it is difficult for the teacher to take care of them one by one individually. The whole class validation in the *neriage* phase is a solution for this constraint. In addition, the Japanese classroom is equipped so that the teacher can control the whole class: the blackboard in front in addition to the large display at the side (see Figure 3). These are the conditions or constraints that afford or hinder collective teaching at the *school level* in terms of the codetermination.

Figure 3: The blackboard and the large display in a Japanese classroom.

Another factor that supports collective teaching is the homogeneity or the idea of equality at the *society level*. In Japan in general, the teacher tries to control the whole class, so that every pupil could learn in the same way. The teaching should not be for a particular learner in the classroom. This is why the teacher shared pupil's solutions even in the research phase in the classroom. The phases, *neriage* and *matome*, as a whole class is presumably the effect of such factor[1]. In contrast, in Switzerland, or even in Europe, there is an idea of individualism and "differentiation" seen as a way of promoting equity. What is necessary for each learner is different, and therefore teacher's individualised intervention is necessary.

Teacher's roles and pupil's roles

At the levels of *pedagogy* and probably *discipline*, we consider that the idea on teaching shared in the teachers' community in each country is one of crucial factors that shapes the teacher's role related to

[1] Highly developed teacher's skill of *bansho* (board writing, see Tan, Fukaya, & Nozaki, 2018) would be also a result of this factor.

the validation in the classroom[2]. In Japan, the national curriculum emphasises students' autonomous and independent learning (MEXT, 2008). And in general, Japanese teachers share more or less the idea that in the *problem solving lesson*, the teacher should not directly validate pupil's answer, and it is rather the role of other pupils. The pupils in our lesson knew well this contract, and there were a few pupils who asked the teacher to validate their answers. In contrast in Swiss teachers' community, *the problem solving tradition* is shared. The important is put more on the solving process than its products. This effect is obvious in the Swiss mathematics textbook (Danalet et al., 1999) which includes only the problem-situations, and no explicit concepts or ideas for pupils to learn. In the Swiss lesson of our project, there was almost no synthesis phase, and the teacher did not take much time for introducing the problem and did not intervene often while solving the problem, since it is important for pupils to manage by themselves autonomously according to the problem solving tradition. What is interesting here is that, the problem solving is a shared idea in both countries. The *problem solving lesson* is often considered as an effective lesson and recommended to the teachers in Japan, and our Japanese lesson was also following more or less the process of such lesson. As the name suggests, the idea of problem solving was involved in the development of lesson organisation in Japan (Hino, 2007). However, the lessons in the two countries are very different in their realisations. The interpretation of problem solving and its further development are therefore very different.

Conclusion

Our research shows that teacher education is under the strong effect of cultural factors. We use a project-based international exchange program as a methodological tool for uncovering cultural factors (conditions and constraints) that shape ordinary lessons of a specific country. These factors can particularly be detected in pre-service teachers' efforts of improving the lesson. Pre-service teachers have a conception of an ideal mathematics lesson developed through their learning experience as a pupil, as a student and also during their pre-service teacher training. Levels of didactical codetermination allow us to identify conditions and constraints of higher levels which are often taken for granted and rarely discussed within the country.

Acknowledgment

This work is partially supported by Research Project of Joetsu University of Education, JSPS KAKENHI (JP15KK0113), and PEERS program (Lausanne University of Teacher Education & Board of Higher Education, State of Vaud).

References

Artigue, M., & Winsløw, C. (2010). International comparative studies on mathematics education: A viewpoint from the anthropological theory of didactics. *Recherches en didactiques des mathématiques, 30*(1), 47-82.

Balacheff, N. (1987). Processus de preuves et situations de validation. *Educational Studies in Mathematics, 18*(2), 147-176.

[2] From the perspective of ATD, the shared idea of teaching could be also considered as a part of theoretical elements of the *didactic praxeology* which is a model of teacher's activity.

Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI Bulletin, 58*, 51-63.

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Trans.). Dordrecht, The Netherlands: Kluwer.

Cai, J., Mok, I. A., Reddy, V., & Stacey, K. (2016). International comparative studies in mathematics: Lessons for improving students' learning. In *International Comparative Studies in Mathematics* (pp. 1-36): Springer.

Cai, J., & Wang, T. (2010). Conceptions of effective mathematics teaching within a cultural context: Perspectives of teachers from China and the United States. *Journal of Mathematics Teacher Education, 13*(3), 265-287.

Chevallard, Y. (2002). Organiser l'étude: Ecologie et régulation. In J.-L. Dorier, M. Artaud, M. Artigue, R. Berthelot, & R. Floris (Eds.), *Actes de la 11e école d'été de didactique des mathématiques* (pp. 41-56). Grenoble: La Pensée Sauvage.

Clarke, D., Emanuelsson, J., Jablonka, E., & Mok, I. A. C. (2006). *Making connections: Comparing mathematics classrooms around the world*. Sense Publishers.

Clarke, D., Mesiti, C., O'Keefe, C., Xu, L. H., Jablonka, E., Mok, I. A. C., & Shimizu, Y. (2007). Addressing the challenge of legitimate international comparisons of classroom practice. *International Journal of Educational Research, 46*(5), 280-293.

Danalet, C., Dumas, J.-P., Studer, C., & Villars-Kneubühler, F. (1999). *Mathématiques 4ème année: Livre du maître, livre de l'élève et fichier de l'élève*. Neuchâtel: COROME.

Hart, L. C., Alston, A., & Murata, A. (Eds.). (2011). *Lesson Study Research and Practice in Mathematics Education*.). New York: Springer.

Hino, K. (2007). Toward the problem-centered classroom: trends in mathematical problem solving in Japan. *ZDM, 39*(5), 503-514.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery*. Cambridge: Cambridge University Press.

Margolinas, C. (2004). *Points de vue de l'élève et du professeur. Essai de développement de la théorie des situations didactiques.* (HDR), Université de Provence - Aix-Marseille I,

MEXT. (2008). *Shōgakkō gakushū shidō yōryō kaisetsu sansū-hen*. [Elementary school teaching guide for Japanese course of study: mathematics]. Tokyo: Tōyōkan.

Shimizu, Y. (1999). Aspects of mathematics teacher education in Japan: Focusing on teachers' roles. *Journal of Mathematics Teacher Education, 2*(1), 107-116.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap. Best ideas from the worlds teachers for improving education in the classroom*. New York: The Free Press.

Stigler, J. W., & Perry, M. (1988). Cross cultural studies of mathematics teaching and learning recent findings and new directions. *Effective mathematics teaching*, 194-223.

Tan, S., Fukaya, K., & Nozaki, S. (2018). Development of bansho (board writing) analysis as a research method to improve observation and analysis of instruction in lesson study. *International Journal for Lesson and Learning Studies, 7*(3), 230 - 247.