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COMBINING COMPUTATIONAL THINKING IN MATHEMATICS: ISSUE OF MODELLING AND EVALUATION

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Researchers acknowledge the importance of computational thinking for all, not only computer scientists. Strong links between computational thinking and mathematics led to the integration of computational thinking into mathematics curricula in many countries around the world. This ongoing research aims therefore to deepen our understanding of what computational thinking is and what the relations between mathematics and computational thinking are when secondary school students solve mathematics problems. This contribution proposes an a priori analysis of a task that was experimented with Grade 8 students, to highlight the interplay between mathematical and computational thinking mobilized in the solving process.

INTRODUCTION

For almost twenty years, computational thinking (CT) draws attention of researchers and educators. Research points out the importance of CT as "universally applicable attitude and skill set everyone, not just computer scientists, would be eager to learn and use" (Wing 2006, p.33). Because of the strong links between CT and solving problems in mathematics (Barcelos, Munoz, Villarroel, Merino & Silveira 2018), CT is part of mathematics curricula in many countries around the world. Indeed, as Wing (2006) claims, "Computer science inherently draws on mathematical thinking, given that, like all sciences, its formal foundations rest on mathematics" (Wing 2006, p.35). Recently, the Swiss education policy considers the teaching and learning of CT as a school priority and sets it as a transversal area in mathematics and in other disciplines. Our research aims at investigating proximities and links of CT with mathematical problem solving skills.

Our research is part of a project that aims to better understand what CT in students' mathematical activity is and to inquire about the links between CT and mathematics. To do this, we first present briefly CT in mathematics and our conceptual framework built on the work of Kallia, van Borkulo, Drijvers, Barendsen, and Tolboom (2021). We then provide elements of an a priori analysis of a mathematical problem from our experimentation. We conclude by highlighting a strong intertwinement between CT and mathematical thinking when students model the problem and when they evaluate their program and their solution.

COMPUTATIONAL THINKING IN MATHEMATICS – LITERATURE REVIEW

The concept of CT is complex and various conceptualizations have been proposed from discipline-based, psychology-based and education-oriented perspectives (Li et al. 2020). While some of these perspectives view CT in close association with computers and programming, others take a position viewing CT "as a model of thinking that is more about thinking than computing" (Li et al. 2020, p.4). In these different perspectives, CT is often defined in terms of "main facets, practices, concepts, components, and dimensions" (Stephens & Kadijevich 2020, p.118).

In mathematics education, researchers focus on the interplay between mathematical thinking and computational thinking (for example, Sneider et al. 2014) and define CT in the mathematics context. Weintrop et al. (2016) define CT for mathematics and science in high school classrooms with a taxonomy consisting of four main categories: data practices, modeling and simulation practices, computational problem solving practices, and systems thinking practices. Barcelos et al. (2018) carried out a wide literature review including articles from 2006 to 2017 on mathematical learning through activities aimed at the development of CT. The authors identify similarities between CT and problem-solving skills. In line with Polya (1945), they consider abstraction defined as a combination of analogical thinking, generalization and specialization, and the capacity to decompose a problem as being crucial abilities to successfully complete problem-solving tasks. Yet, Stephens and Kadijevich (2020, p.119) claim that "studies explicitly linking CT and learning mathematics are rather rare, mostly dealing with areas that are traditionally connected to programming, for example, numbers and operations, algebra, and geometry".

Among the different approaches characterizing CT in mathematics education, we adopt the conceptual framework of Kallia et al. (2021). The authors, drawing on an extensive literature review of 56 papers complemented by a Delphi study, highlight three key aspects of CT in mathematics education:

(1) problem-solving as a fundamental goal of mathematics education in which computational thinking is embedded;

(2) thinking processes that include (but not limited to) *abstraction, decomposition, pattern recognition, algorithmic thinking, modelling, logical* and *analytical thinking, generalization* and *evaluation* of solutions and strategies;

(3) phrasing the solution of a mathematical problem in such a way that it can be transferred / outsourced to another person or a machine (transposition) (p. 179).

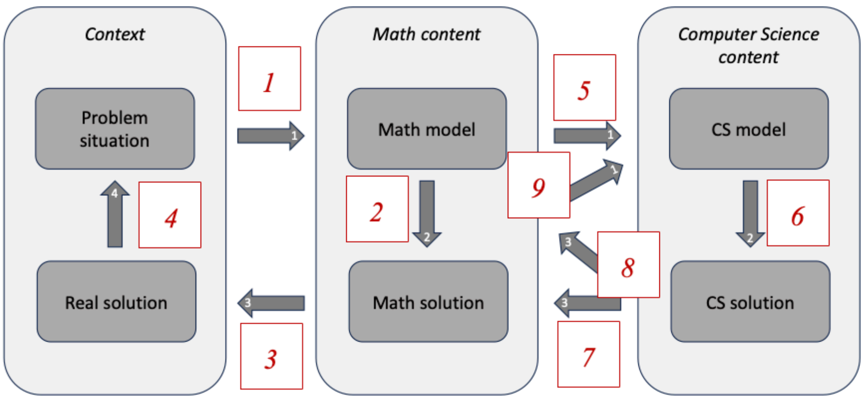

Figure 1: Computational thinking in mathematics based on Kallia et al. (2021, p.3)

Conceptual framework of CT in mathematics elaborated by Kallia et al. (Fig. 1) highlights four categories of cognitive activities (see numbered arrows): (1) translating a situation into mathematical or computational model (modelling, abstraction and pattern recognition); (2) reasoning and working within mathematics and computer science; (3) translating the result back into the context (involving generalization); and (4) verifying if this really solves the real-world problem adequately (evaluation). Based on this framework, we address the following research question: how does the interplay between mathematics and computational thinking contribute to solving a mathematical task? For the purposes of our research, we have numbered the arrows (*1-9*) in the Figure 1 and we refer to this numbering.

EXAMPLE OF COMBINING MATHEMATICS AND COMPUTATIONAL THINKING

In collaboration with a secondary mathematics teacher, we designed a task inspired by the famous 'wheat and chessboard' problem[1] (Fig. 2) to be solved with Scratch. The original task, found in the chapter on real numbers in Grade 8 textbook for French-speaking Swiss schools, is related to powers. The students are thus expected to express the number of rice grains on the n-th square of the chessboard as 2^{n-1} (pattern recognition and generalization); the expected answer to the question 1 is thus 2^{63}. This solution relies on *mathematical modelling* (see arrows *1-4* in Fig. 1). Indeed, the pattern recognition and generalization at stake characterize also algebraic thinking (Radford, 2018).

> A legend claims that the inventor of chess is Sessa, an Indian sage. He would thus have succeeded in distracting a king who, wanting to thank him, offered him to choose a reward himself.
>
> Sessa: I would like to have a bit of rice.
> King: Perfect, Sessa. How much rice do you want?
> Sessa: Look, you put one grain on the first square, then two on the second, four on the third, eight on the fourth and so on until the sixty-fourth square, doubling the number of grains on each subsequent square.
>
> The king was surprised and amused by such a modest request.
>
> In your opinion, is this reward that modest?
>
> 1) How many grains did the king have to place on the last square of the chessboard?
>
> **To go further**
>
> 2) How many grains did the king have to place on all the squares of the chessboard?
> 3) Knowing that on average, there are 35000 grains of rice in a kilogram of rice, how many kilograms does it represent? Compare to the world rice production knowing that in 2021, 499 million tones of rice have been produced.

Figure 2: The task proposed to Grade 8 students (Batteau & Trgalová 2023)

To prompt the recourse to CT, we first ask students to find the exact number of rice grains on the last chessboard square (question 1) and then the total number of grains on the whole chessboard (question 2), that is needed for the comparison with the world rice production (question 3). We also assume that the exact number of grains (9 223 372 036 854 775 808) is more striking than 2^{63} whose rough size might not be obvious for the students. Using a calculator, the students would find a number in a scientific notation (9.223372037e+18). Using Scratch would thus appear as a necessity to calculate the exact number of the rice grains.

Since there is no power operator in Scratch, the students need to define a variable (e.g., rice), set it to 1 and use a loop to double the value of the variable rice 63 times, either by adding the same value to the variable value or by multiplying the variable value by 2, yielding different *CS models* (see arrow *5* in Fig. 1). The outcome of the Scratch program (see arrow *6* in Fig. 1) is 9 223 372 036 854 776 000. At this stage, a verification of the program should be done by comparing the mathematical model and

[1] https://en.wikipedia.org/w/index.php?title=Wheat_and_chessboard_problem&direction=next&oldid=775634591

the CS model (see arrow *8* in Fig. 1), for example by running the program for smaller powers of 2 and comparing the results with those obtained by hand. The correctness of the program however does not guarantee the correctness of the output (see arrow *7* in Fig. 1). Validating the output requires coming back to the mathematical model (*8* in Fig. 1): the observation of the first powers of 2 – 1; 4; 8; 16; 32; 64; 128… – allows seeing that no power of 2 can have 0 as the last digit.

CONCLUSION

Our example highlights the interplay between mathematics and computational thinking when solving a mathematical task. First, a mathematical model for solving a given task needs to be built. Since the mathematical solution obtained is not convenient, a solution can be formalized to be outsourced by a computational environment, which requires elaborating a CS model. The verification of the CS model, to ensure that it achieves its goal without any bug, relies on a comparison of mathematical and CS models. The validation of the CS solution (the program output), to ensure its correctness, mobilizes mathematical knowledge as well. Thus, in the task presented above, mathematics provide a model that is subsequently formalized to yield a CS model. Verification of the model and validation of the CS solution mobilizes mathematical knowledge. Further studies are needed to explore such interplay in other problems and to clarify the skills involved in solving mathematical problems and their relations with mathematics and computer science.

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