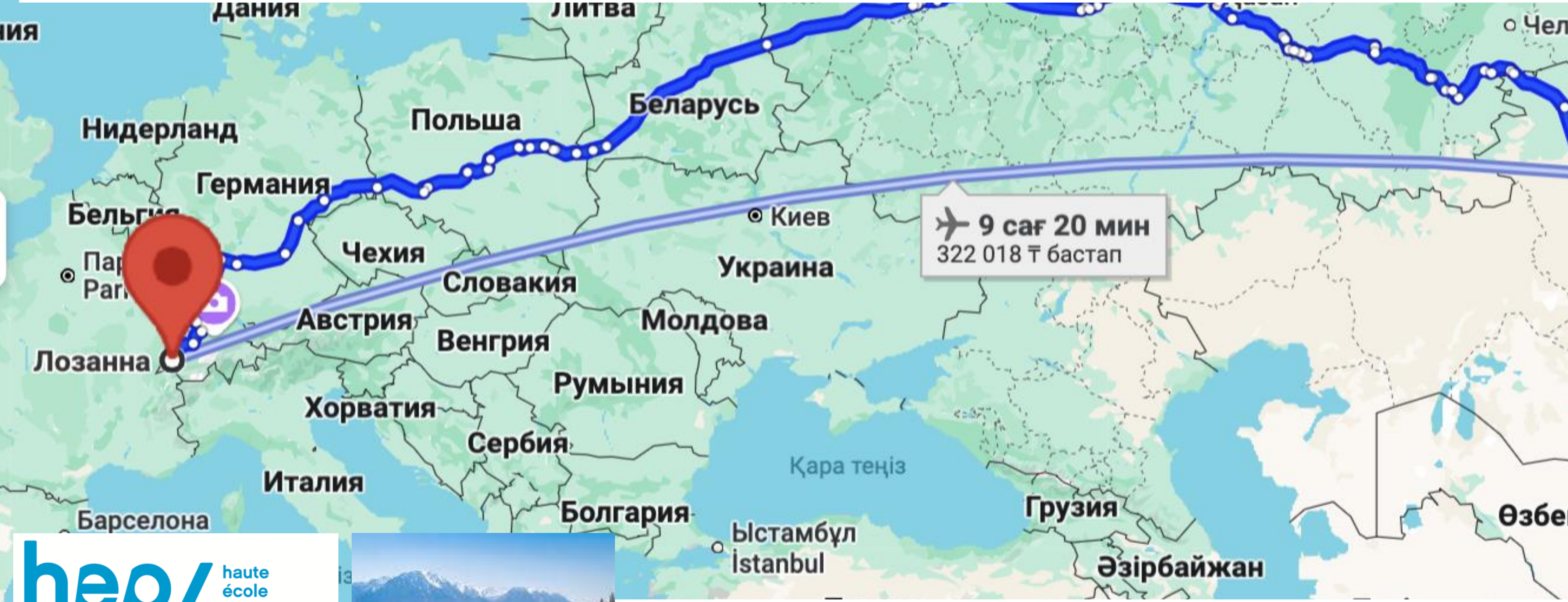
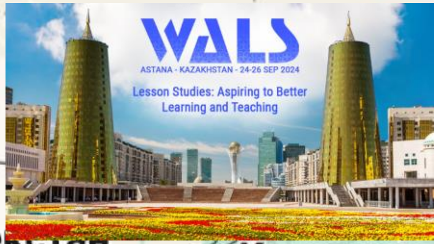
From collective preparation to research lesson focused on students computational thinking in mathematics problem solving





hep/ haute école pédagogique vaud

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Plan

- Computational thinking in maths
- Framework of the study
- Methodolgy
- Elements of analysis
- Conclusion/perspectives

What is computational thinking in mathematics?

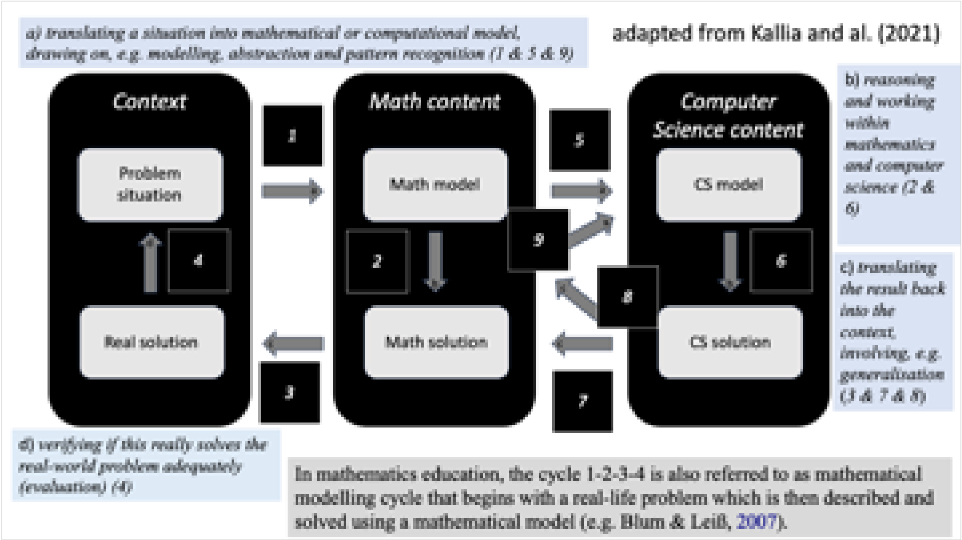
CT in mathematics is a model of thinking that includes skills of abstraction, problem decomposition, data analysis and interpretation, pattern recognition, and modeling aiming at solving a mathematical problem by formalizing a solution in a way that it can be outsourced by a machine. (Kallia et al., 2021).

Framework of the study

- **Context**
  - A team of two Swiss mathematics education researchers has chosen to work on computational thinking in mathematical problem solving in the form of a lesson study. This team of researchers collaborated with one mathematics teacher in 2022/2023 and with two teams of two mathematics teachers in 2023/2024 in two Swiss secondary schools.
- **Question**
  - The main question guiding our research is whether mobilizing computational thinking to solve mathematical problems enables students to develop a different/better understanding of the mathematical concepts at stake (in comparison with mathematical teaching without such thinking) or to overcome epistemological obstacles linked to the concepts at stake.

Framework of the study

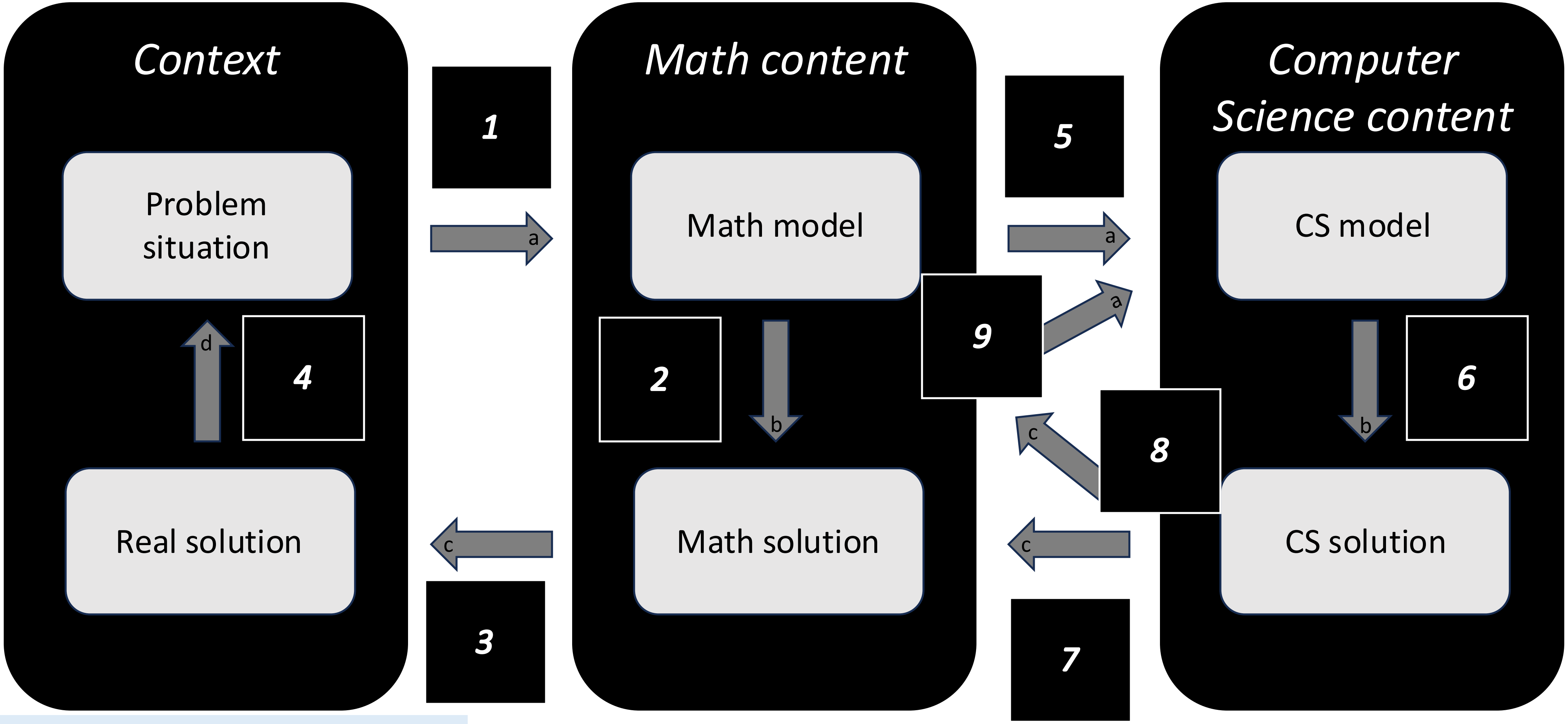
- Theoretical framework, to analyse computational thinking, adapted from Kallia et al. (2021)



a) translating a situation into mathematical or computational model, drawing on, e.g. modelling, abstraction and pattern recognition (1 & 5 & 9)

adapted from Kallia and al. (2021)

b) reasoning and working within mathematics and computer science (2 & 6)



c) translating the result back into the context, involving, e.g. generalisation (3 & 7 & 8)

d) verifying if this really solves the real-world problem adequately (evaluation) (4)

In mathematics education, the cycle 1-2-3-4 is also referred to as mathematical modelling cycle that begins with a real-life problem which is then described and solved using a mathematical model (e.g. Blum & Leiß, 2007).

Methodology

Qualitative research methodology supported by an adapted lesson study process: a collaborative and iterative process of design and implementation of mathematical tasks, involving researchers and teachers (Batteau & Clivaz, 2016; Psycharis et al., 2020)

Group of 2 researchers and 2 secondary math teachers

A priori/a posteriori analysis using the Kallia et al.'s model

2023 September

Session 1
General discussion about lesson study, collaboration, computational thinking, choice of mathematical subject (geometry: regular polygones)

November

Session 2
Choice of a mathematical task

2024 January

Session 3
Analysis of the task, preparation of the sequence of lessons (choice of Scratch functions)

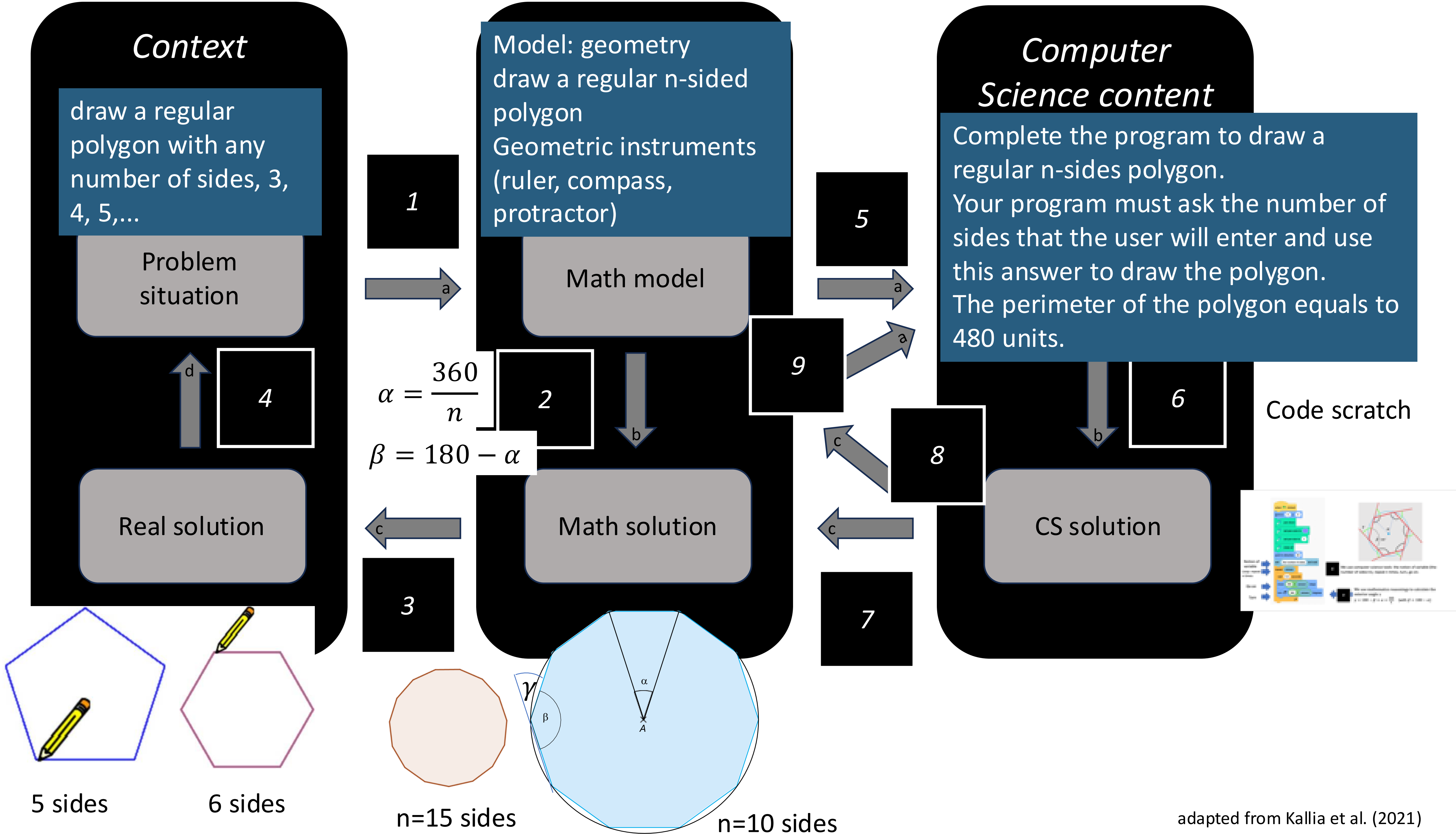
March

Session 4
(11.03)
Preparation of the RL

Research lesson
(27.03)

Session 5
Discussion post-RL

Elements of *a priori* analysis



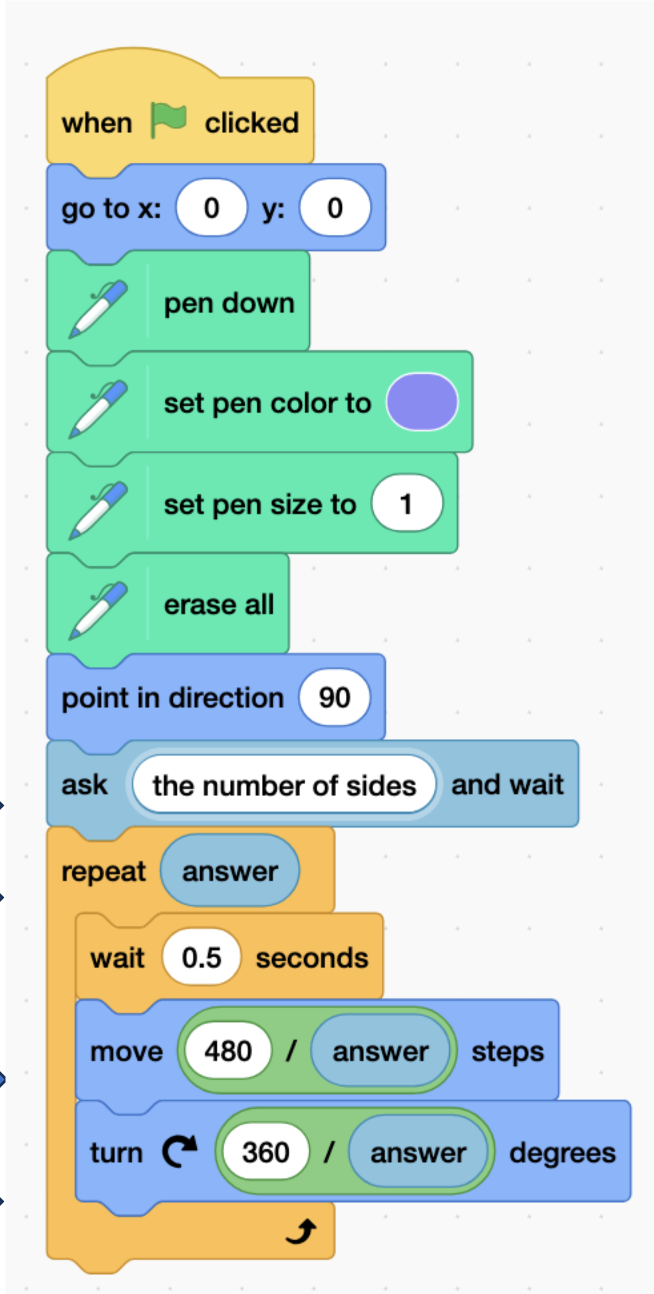
adapted from Kallia et al. (2021)

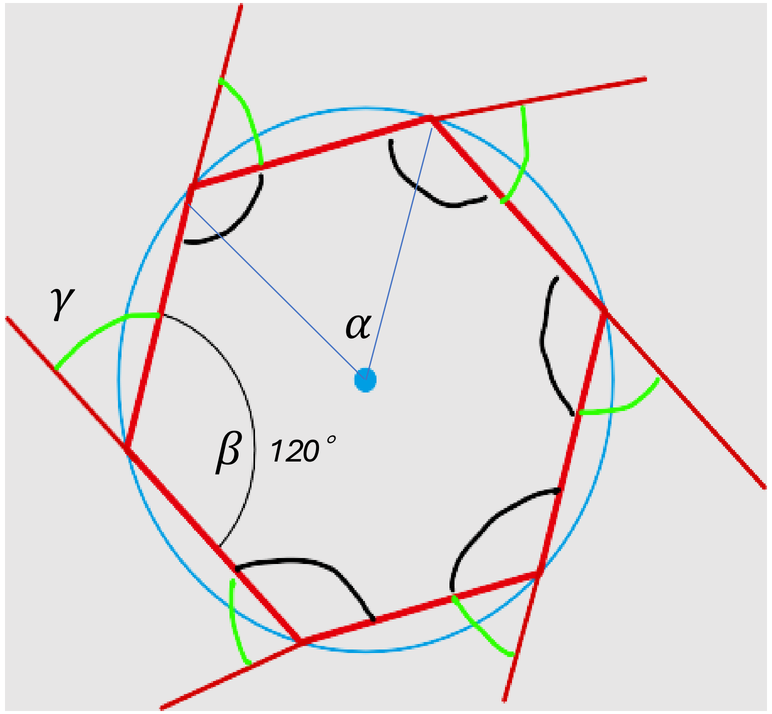
Notion of variable

Loop: repeat n times

Go on

Turn





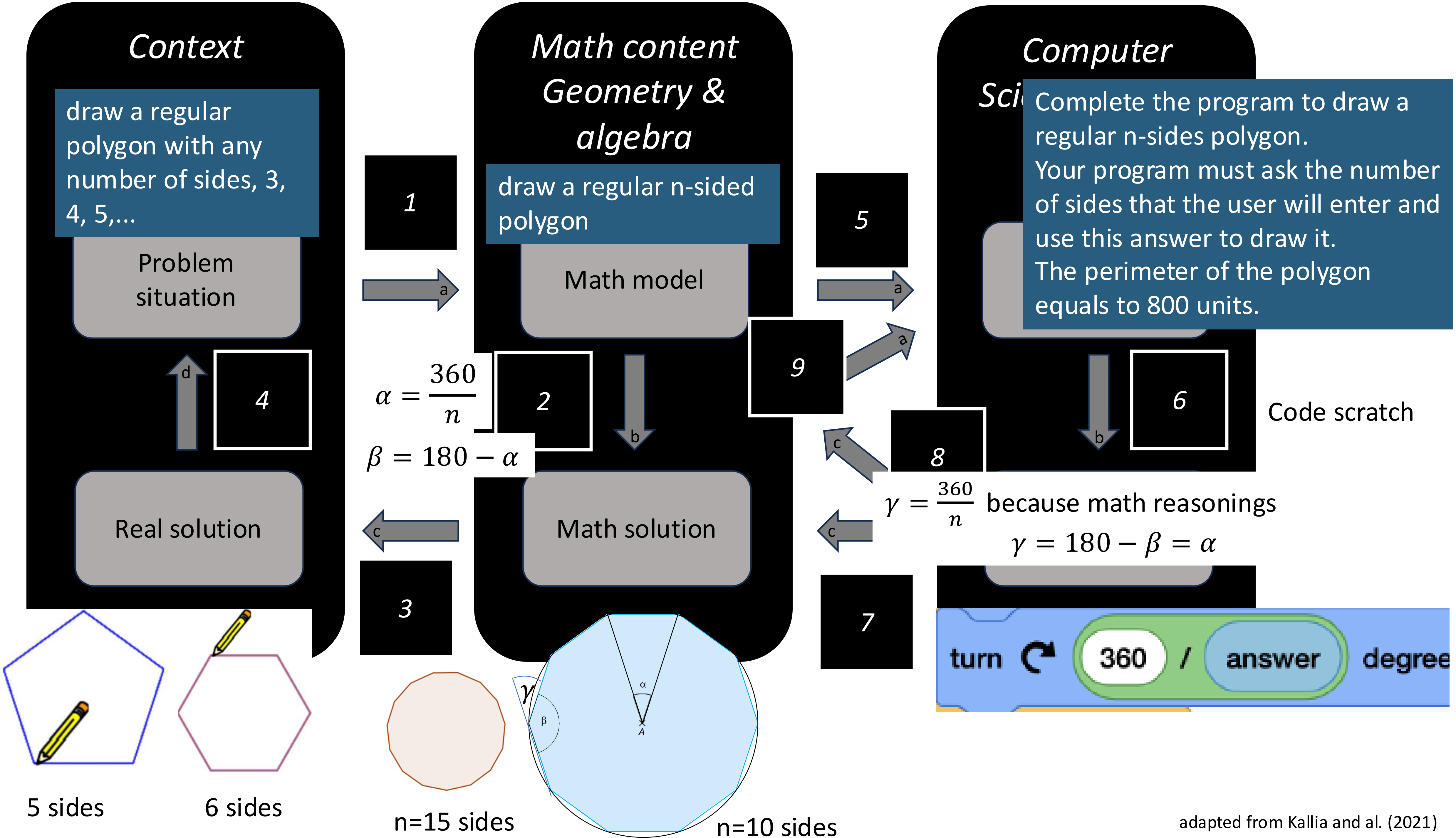
6

We use computer science tools: the notion of variable (the number of sides=n), repeat n times, turn, go on.

9

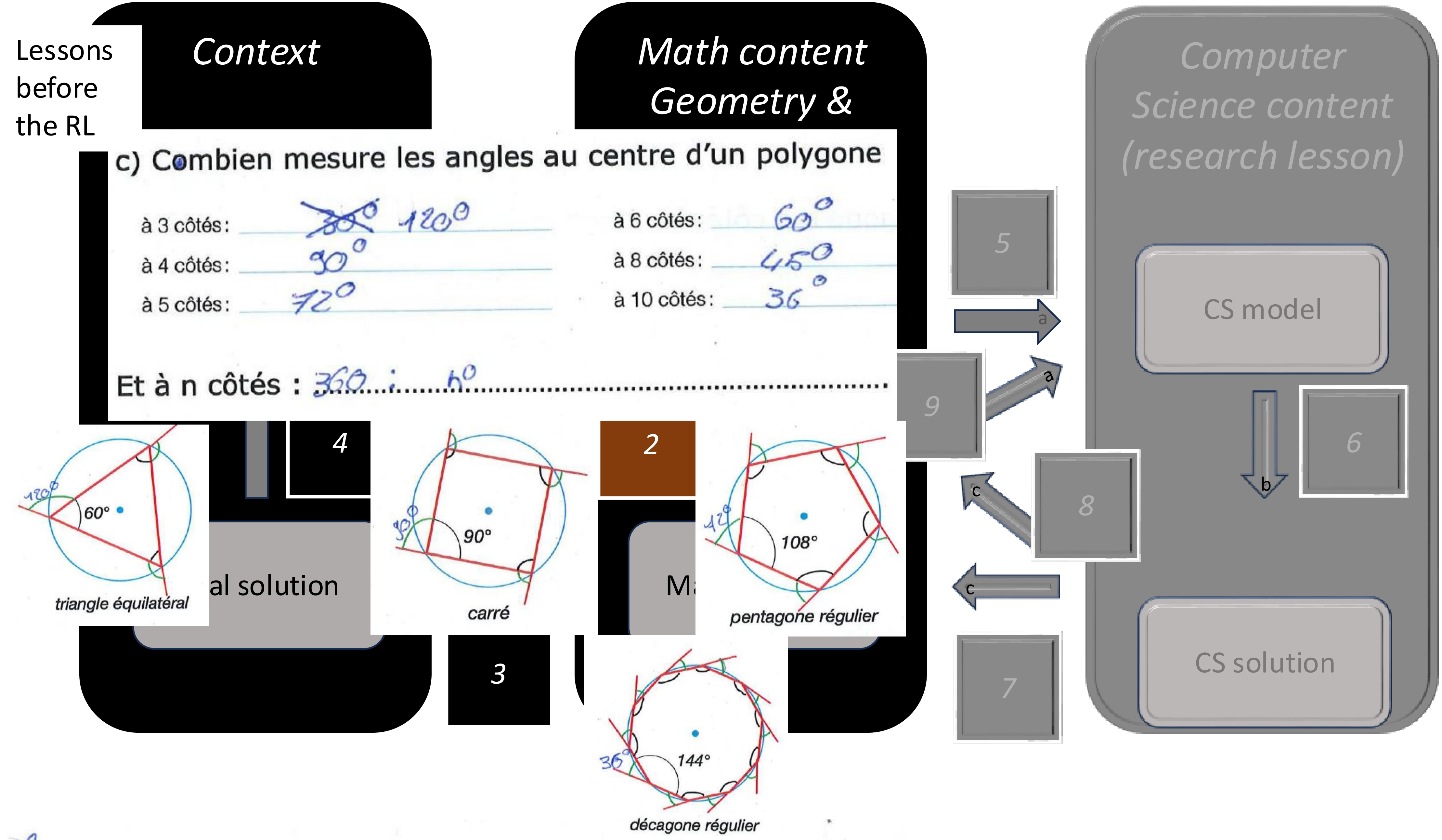
We use mathematics reasonings to calculate the exterior angle γ

$$\gamma = 180 - \beta = \alpha = \frac{360}{n} \quad (\text{with } \beta = 180 - \alpha)$$



adapted from Kallia and al. (2021)

Elements of *a posteriori* analysis



Les angles au centre sont les mêmes que ceux à l'extérieur

angles at the center equal exterior angles. Pattern recognition (algebraic thinking)

adapted from Kallia et al. (2021)

During a whole-class discussion of the RL

Perimeter of the regular polygon=480 units

Complète le tableau sachant que le périmètre des polygones réguliers est de **480 pas**

Nom du polygone régulier Name of the regular polygon	**Nombre de côtés** Number of sides	**Mesure d'un côté** Length of a side	**Angle extérieur =** Exterior angle γ ....
Triangle équilatéral Equilateral triangle	3	480: 3	120
Carré square	4	480: 4	90
Pentagone pentagon	5	480: 5	72
Hexagone hexagon	6	480: 6	360: 6
Octogone octagon	8	480: 8	360: 8
Décagone decagon	10	480: 10	360: 10
Dodécagone dodecagon	12	480: 12	360: 12
Polygone à n côtés n-sided regular polygon	n	480: n	360: n

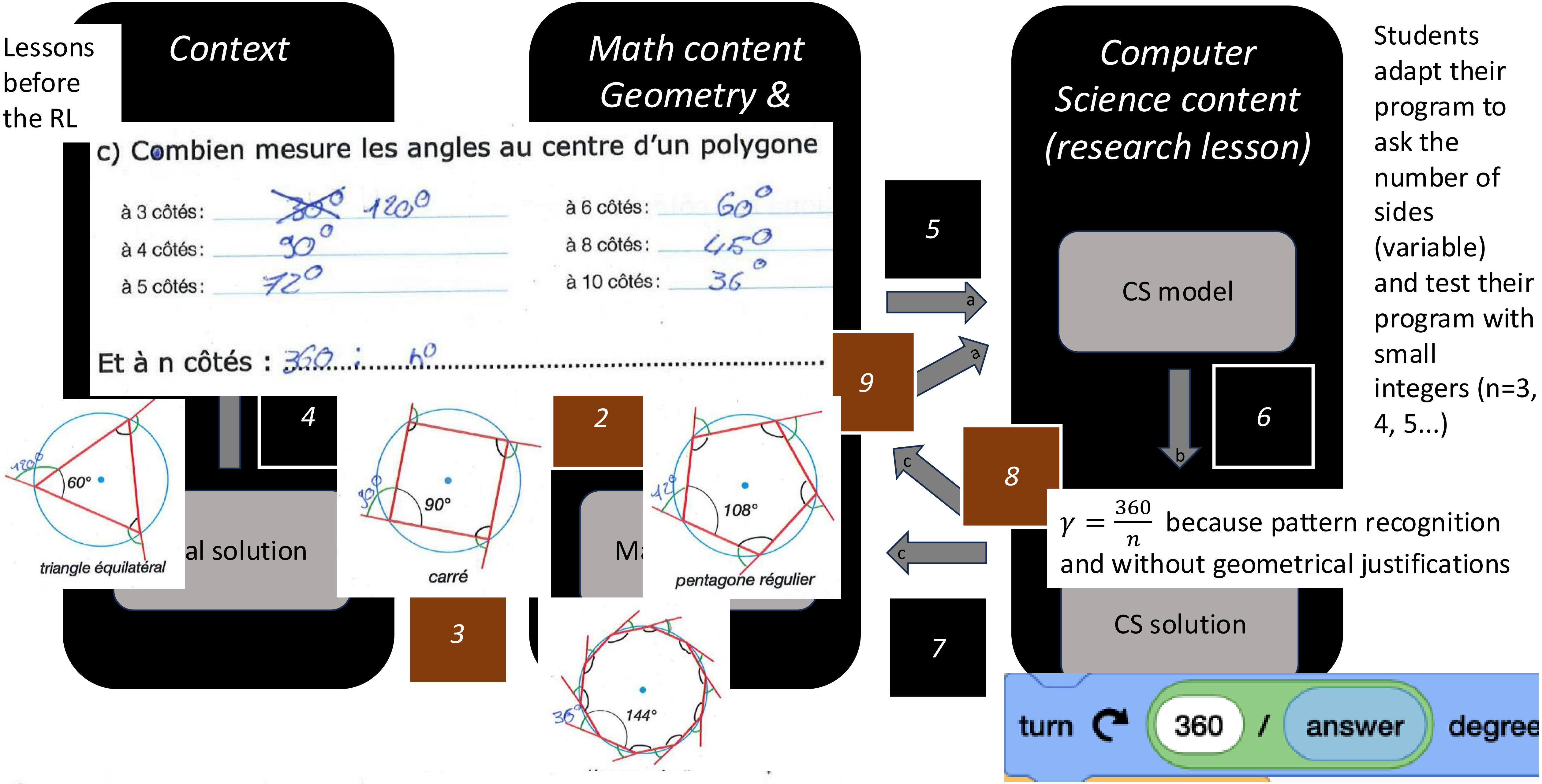
Pattern recognition divided by the number of sides

Generalisation $\gamma = \frac{360}{n}$

they test the program for n=3, 4, 5, … to verify their program.

Length of a side= $\frac{perimeter}{n} = \frac{480}{n}$

Students do not explain why $\gamma = \frac{360}{n}$ using geometrical reasonings with the sum of the angles in a triangle. They explain this equality (Interior angle= exterior angle) with pattern recognition and generalization during lessons before the RL (ex. 4c & 5), and during the whole-class discussion of the RL.



Les angles au centre sont les mêmes que ceux à l'extérieur

angles at the center equal exterior angles. Pattern recognition (algebraic thinking)

adapted from Kallia et al. (2021)

Conclusion & perspectives

- One objective of this LS was to mobilize computational thinking when students solve mathematical problem. Students mobilize their computational thinking in order to program drawing of a n-sided regular polygon with Scratch (pattern recognition, generalization, decomposition, abstraction, modeling, use of Scratch to code a program to draw a n-sided polygon), but they cannot justify why the angle at the center is equal to the exterior angle neither in the math nor in computer science contents. They observe it with pattern recognition and generalization (but do not prove it).
- Students use a lot of mathematical knowledge to solve the problem: perimeter, regular polygons, angles, interior/exterior/center angle in a regular polygon, generalization (to n sides), solving problem skills and in computer science: the Scratch environment, the notion of variable, the notion of loop, the use of different geometric functions on scratch (move, turn, pencil)
- But to answer to our main question, we can not declare if students developed a better/different understanding of the mathematical concepts at stake (regular polygons, notion of angles and so on, in comparison with mathematical teaching without such thinking) or to overcome epistemological obstacles linked to the concepts at stake.

Conclusions & perspectives

From collective preparation to research lesson focused on computational thinking students in mathematics problem solving

- During collective preparation, we need to highlight all the expected reasonings in mathematical content and computer science content. The research lesson allows to highlight the students' reasonings to solve problems in mathematic content and in computer science content. The analysis of the RL with the Kallia et al.'s model allows us to highlight the difficulties and the nodes in the teaching of problem solving with CT.
- We used the model of Kallia et al. during our analysis after the research lesson and not to prepare the lesson. Thus, the use of Kallia et al. model will allow us
  - to work with teachers during sessions of lesson study to anticipate the specific difficulties of mathematical problem solving in math content and computer science content.
  - to pay attention to all the transitions between the context, the mathematical content and the computer science contents.

Thank you!
Merci!
Рахмет!