

**International Symposium
Elementary Mathematics Teaching**

Prague, the Czech Republic
Charles University, Faculty of Education

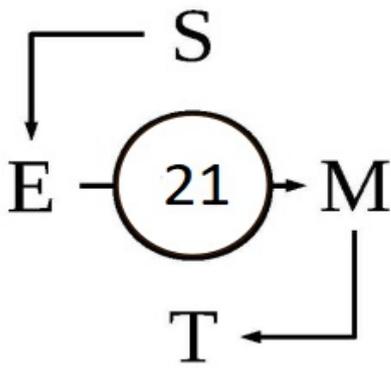
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Proceedings

**Broadening experiences
in elementary school mathematics**

Edited by Jarmila Novotná and Hana Moraová

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International Symposium Elementary Mathematics Teaching

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Since all papers and other presentations here are presented in English, which is not usually the first language of the presenters, the responsibility for spelling and grammar lies with the authors of the papers themselves.

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WORKSHOPS

A MATHEMATICAL PERSPECTIVE ON UNITS IN ARITHMETIC TEACHING AND LEARNING: A COMPARISON OF FRENCH, SWISS, AND JAPANESE CONTEXTS

Christine Chambris  and *Valérie Batteau*  

Abstract

Several theoretical perspectives have pointed out the importance of units in teaching and learning elementary arithmetic. Adopting an institutional perspective, the workshop will focus on some features of arithmetic in three educational systems that consider units differently, and that provide different experiences for multiplicative reasoning. This will be preceded by analyses of excerpts of an 18th century arithmetic textbook that shaped decades of teaching, at least in France.

Keywords: units, magnitudes, arithmetic, place value, multiplication

The workshop aims to present recent and in progress research that focuses on units, in a mathematical perspective. This mathematical perspective on units enriches the one regarding China presented by Ma (Ma & Kessel, 2018) on the occasion of 23rd ICMI Study. It extends prior works on the role of magnitudes in arithmetic prior to the reform of the New Math (Chambris, 2017) in some western countries. It also resonates with design studies that consider units from an

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instructional design perspective (Visnovska, Cortina & Vale, 2019) and from a psychological perspective (Saenz-Ludlow, 2019). It also provides a new look on teaching in the Japanese educational context (Batteau, 2019).

The units in arithmetic have long been studied in psychology, especially for learning arithmetic (see Tzur, 2019 for a recent summary). In particular, the “composite unit –a unit the mind produces by putting together smaller units (e.g., 5 is a unit made of five 1s, or 3 and 2)” (Tzur, 2019, p. 29) –appears to be an important mental operation. Indeed, concept of number as a “composite unit” is a keystone for progress in arithmetic (Tzur, 2019). It seems this approach implies that, in mathematics as a discipline, nothing distinguishes a number as a composite unit and at a less advanced stage of development.

In the mathematical perspective, units appear in measurement as values of magnitudes. However, specific properties of units intervene when units are used in arithmetic. These mathematical features are linked with magnitudes. The units for arithmetic are used in the Japanese context but –at present time- they are not taught as such in several other educational contexts, at least in France (Chambris, 2018) and in Switzerland (Batteau & Clivaz, 2016). Nevertheless, they provide powerful means for reasoning, and in particular for multiplicative reasoning. For this reason, this perspective on units can be seen as a means to broaden experiences in elementary school mathematics.

Using successive approaches, we point out:

Session 1: The mathematical roots of units in arithmetic and some facets of the roles of units in various sub-topics of arithmetic such as place value, fractions, and multiplication. Participants will analyze excerpts of a French reference book for teaching arithmetic of the 18th century and additional information regarding notably the mathematics of the time will be given.

Session 2: The educational contexts in which units are somehow missing in classes. In the French and Swiss contexts, curricular data in arithmetic topics provide many examples where misunderstandings / problems that involve units can be seen. Participants will analyze a sample of data from classes and/or textbooks, in connection with the topic of place value.

Session 3: The Japanese context in which units are in the foreground in arithmetic and measurement of magnitudes teaching. Curricular data that involve learning of multiplication and measurement will be considered. Participants will analyze data from Japanese classes in the context of lesson studies and/or textbooks and/or reference books for teaching.

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MAKE ROOM FOR DAMC

Vesife Hatisaru 

Abstract

This workshop introduces the Draw a Mathematics Classroom (DAMC) instrument, and its associated rubrics developed by the presenter. The DAMC provides teachers with a tool to see the mathematical practices in the classroom through the eyes of students. Teachers can use DAMC to access and become aware of their students' views about mathematics learning experiences and use such understanding as a basis for reflecting on their teaching. As a result, teachers may give thoughts on (if any) modifying chiefly expository teaching strategies and embracing variety in teaching methods.

Keywords: Draw a Mathematics Classroom Test, image of mathematics, learning practices, teaching practices

Introduction

In a previous study, I explored a large group of 1284 Turkish middle school students' (aged 11 to 14) images of mathematics through examining their *Draw a*

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